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A dynamic inhomogeneous latent state model for measuring material deprivation

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Summary. Material deprivation can be used to assess poverty in a society. The status of poverty is not directly observable, but it can be measured with error for instance through a list of deprivation items. Given two unobservable classes, corresponding to poor and not poor, we develop a time inhomogeneous latent Markov model which enables us to classify households according to their current and intertemporal poverty status, and to identify transitions between classes that may occur year by year. Households are grouped by estimating their posterior probability of belonging to the latent status of poverty. We then estimate an optimal weighting scheme, associated with the list of items, to obtain an optimal deprivation score. Our score is arguably better at predicting the poverty status than simple item counting (equal weighting). We use the longitudinal component of the European Union statistics Survey on Income and Living Conditions for evaluating poverty patterns over the period 2010–2013 in Greece, Italy and the UK.

Keywords: European Union Survey on Income and Living Conditions; Latent Markov models; Material deprivation

1. Introduction

A widely agreed concept of material deprivation, which inspires the principles of the European Union (European Commission, 2004), refers to a lack of

> 'the material standards of diet, clothing, housing, household facilities, working, environmental and locational conditions and facilities which are orderly available in their society'

(Townsend (1987), page 140). It is a relative concept, regarding the position that is occupied by individuals within the society that they belong to, that traces back to the early stages of economics. As an aside, Smith (1776) described a crucial component of social life as the ability to appear in public without shame with these words:

> 'By necessaries I understand not only the commodities which are indispensably necessary for the support of life, but whatever the custom of the country renders it indecent for creditable people, even of the lowest order, to be without. A linen shirt, for example, is, strictly speaking, not a necessary of life. The Greeks and Romans lived, I suppose, very comfortably though they had no linen. But in the present times, through the greater part of Europe, a creditable day-laborer would be ashamed to appear in public without a linen shirt, the want of which would be supposed to denote that disgraceful degree of

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poverty which, it is presumed, nobody can well fall into without extreme bad conduct. Custom, in the same manner, has rendered leather shoes a necessary of life in England. The poorest creditable person of either sex would be ashamed to appear in pubic without them.'

Material deprivation is a consequence of poverty, where poverty is the lack of resources that are deemed necessary to reach an acceptable standard of living in the society (Townsend, 1987; Gordon, 2006). Measuring deprivation is seen as a 'direct method' of poverty measurement (Sen, 1981), since it focuses on the (enforced) actual lack of a combination of commodities or activities, i.e., still in Sen's words, on the number of the individual functioning failures regarding material living conditions. This approach has been often compared with the prevailing approach for identifying the poor, i.e. the 'income (or consumption) method'. In cross-sectional surveys it is recommended to measure both income and deprivation to identify the poor (Townsend and Gordon, 1991); but also in longitudinal surveys the potential presence of measurement errors has led to arguing in favour of dealing with both methods separately and, eventually, of evaluating their mismatch (Whelan *et al*., 2004).

Measurement of material deprivation has generally followed the 'counting approach', i.e. a parsimonious way of classifying a society according to the number of 0–1 deprivation indicators (deprivations) that lead to a deprivation score. A different approach is based on welfare-based measures allowing analysts to give normative content to the measurements of deprivation. See Atkinson (2003) for an axiomatic comparison between the counting approach and the social welfare function approach. An individual score of deprivation results from the (possibly weighted) sum of the dichotomous indicators. In the most simple case of equal item weights, deprived individuals are defined as those lacking at least a certain number of items. Two individuals with the same deprivation score are treated equally, even though they do not necessarily lack the same items. The cut-off, the list of items and their associated weights have been a matter of concern and dispute, since they can affect the results and the consequent policy.

An increasing stream of literature relates multiple-deprivation measurement to latent variable models (Whelan and Maître, 2006; Machado *et al*., 2009; Deutsch *et al*., 2015; Najera Catalan, 2017) based on the acceptance that observed heterogeneity in society is due to unobserved conditions. In that respect, a society can be clustered into groups, say poor and not poor, where poverty is the latent variable whereas deprivations are the manifest observed variables (e.g. lacking shoes, clothes or food) that can be used to assess population heterogeneity empirically. For a general discussion about this conceptualization of latent variable models see Skrondal and Rabe-Hesketh (2004).

Another important issue is the distinction between current and persistent deprivation or poverty. Snapshots of who is poor in a given year (cross-sectional or current poverty) provide an incomplete picture of the prevalence of poverty, since it leaves open the question of whether individual poverty is a persistent or transitory phenomenon. The vast majority of empirical studies in this domain assesses static deprivation or poverty, whereas the measurement of its intertemporal dimension, capturing extreme forms of persistent deprivation or poverty and mobility into and out of, is still lively debated (see Bossert *et al*. (2012, 2014)).

Given the availability of reliable longitudinal data in poverty research, our main goal is to develop a *dynamic* latent state model that can classify individuals (or households) according to their unobserved poverty status from their observed current and intertemporal deprivations and to estimate movements into and out of poverty during the whole observation period.

Our strategy treats the observed pattern of presence or absence of deprivations as a multidimensional binary outcome whose joint distribution is determined by the membership of a latent state. The latent status is represented by a latent process that is assumed to follow a first-order Markov chain with a finite number of states (poverty and not poverty) and assigns individuals (or households) to their latent status by posterior probabilities of membership. In this dynamic perspective, the probability of being *persistently* poor is estimated as the joint probability of being poor over the whole period and transitions between classes (poor and not poor) that may occur year by year can be estimated. Implicit in this approach is a more flexible classification that reduces the arbitrariness due to the choice of lacking deprivation items that are necessary for defining the threshold between poor and not poor, continuing the efforts of Pittau and Longford (2006), Anderson *et al.* (2014), Anderson, Pittau and Zelli (2016) and Anderson, Farcomeni, Pittau and Zelli (2016).

Formally, our model belongs to the class of latent Markov models for longitudinal data (Bartolucci *et al.*, 2013, 2014; Farcomeni, 2015) and it is a generalization (to the case of weighted likelihood by sampling weights) of the multivariate marginal approach, initially proposed by Bartolucci and Farcomeni (2009). (For an introduction to such models see Collins and Lanza (2010) and Bartolucci *et al.* (2013), chapter 1.) Our latent variable actually follows a *time inhomogeneous* Markov chain, which is fully specified by an initial distribution and time-dependent transition matrices (Bartolucci *et al.*, 2009). A similar switch of perspective has also been proposed in other contexts; see for example Paas *et al.* (2007), Bartolucci *et al.* (2007) and Lagona *et al.* (2015).

An innovative feature of our study, from a methodological perspective, is a rationale for dimension reduction of the binary outcome. The resulting one-dimensional outcome, i.e. an overall deprivation index, is based on an optimally weighted sum and can be used as a score for measuring material deprivation and subsequent thresholding.

The paper is organized as follows. The next section briefly recalls the counting approach to measure material deprivation introduced in the literature for cross-sectional and longitudinal data. Section 3 sketches our model, highlighting its potential advantages in analysing the prevalence of poverty in a society, and the estimation strategy. Section 4 presents the data and the main empirical results. Section 5 finally concludes.

2. The counting approach

The counting approach that is conventionally employed to identify the deprived population can be summarized as follows. Assume that we have N agents (households or individuals) of a population, R items of material deprivation (deprivations) and T periods of time. For each individual $i \in \{1, \ldots, N\}$, for each $t \in \{1, \ldots, T\}$ and for each item $r \in \{1, \ldots, R\}$, we observe whether the individual is deprived in item r at time t. A binary deprivation variable Y_{itr} is defined, where $Y_{itr} = 1$ means that individual i at time t is deprived in item r.

An individual can be identified as deprived if the number of lacking items is above a fixed value, say K, with $1 < K < R$. This measure can be extended by assigning different weights to different deprivations. The weights may reflect the normative statement that policy makers attach to each item or the preferences that are expressed by the society on the most detrimental deprivations in that period. Decancq and Lugo (2013) presented an overview and discussed the weight assignment in the context of multi-dimensional wellbeing. The deprivation score of each agent results in the weighted counting value: $\bar{Y}_{it} = \Sigma_{r=1}^{R} \tau_r Y_{itr}$ where τ_r is the weight assigned to the deprivation item r. More generally, weights might be time dependent. Finally, a deprivation cut-off λ_t is set for inclusion and exclusion purposes to establish who is deprived and who is not, i.e. $d_{it} = \mathbb{I}(\bar{Y}_{it} \geqslant \lambda_t)$, where $\mathbb{I}(\cdot)$ denotes the indicator function.

The adoption of a longitudinal perspective enables identification of individuals who suffer from persistent or chronic material deprivation. The longitudinal measures that have been proposed in the literature can broadly be comprised of two main approaches. The first is based on a measure of individual intertemporal material deprivation, which for each individual can be formalized as a weighted mean of the deprivation scores over the T periods. The rationale is that longer breaks between deprivation spells reduce individual intertemporal deprivation whereas longer deprivation spells increase individual intertemporal deprivation (Bossert *et al.*, 2014; Dutta *et al.*, 2013; Mendola *et al.*, 2011). The second route is to define persistent deprivation based on the total amount of time that an individual spends in deprivation over the whole period. This is called the 'spells' approach (Bane and Ellwood, 1986), which is based on a total duration cut-off.

3. Methodological framework

3.1. A sketch of the model

Individuals belong to the latent state of poverty with a probability that depends on the presence or absence of a specific combination of deprivation items. More formally, the outcome for the ith individual at time t is the R-dimensional configuration $Y_{it}=(Y_{it1},\ldots,Y_{itR})$. The R-dimensional outcome measures, with error, a binary latent variable U_{it} which represents for each individual i an indicator of being in the poverty status in the simplest case of $k=2$, and an indicator of being in the jth latent group at time t, $j=1,\ldots,k$, in the more general case. Subjects are allowed to move from one latent state to another between each measurement occasion; hence U_{it} is not necessarily constant over time. In what follows we assume that the ith subject has been measured at times $1,\ldots,T_i$, with $T=\max_i T_i$, and that missing measurements are not informative. In our analysis $T_i=T=4$.

Although throughout we assume $k=2$, we here give formal definitions for a general k. When the task is clustering or identification, as with the latent poverty status, users often work with a small and prespecified k. In our case, one would always fix $k=2$ or $k=3$ (as Eurostat does). However, in cases in which the main interest lies in removing bias arising from unobserved heterogeneity, possibly larger values can be used. A formal choice of k usually proceeds through minimization of information criteria.

We assume that Y_{itr} is independent of previous measurements and other outcomes conditionally on the unobserved discrete latent variable U_{it}. This is the usual assumption of *local independence* and guarantees that all information agent i at time t is summarized by U_{it}. We assume that U_{it} has support points $j=1,\ldots,k$. The so-called manifest distribution can then be summarized as

$$\Pr(Y_{itr}=1|U_{it}=j)=p_{jr}, \qquad j=1,\ldots,k, \tag{1}$$

for $t=1,\ldots,T_i$.

A first-order Markov chain for the latent variable U_{it} is specified with an initial distribution

$$\Pr(U_{i1}=j)=\pi_j, \tag{2}$$

where $\Sigma_j\pi_j=1$, and an *inhomogeneous* transition distribution of the form

$$\Pr(U_{it}=j|U_{i,t-1}=h)=\pi_{jth}, \qquad t=2,\ldots,T_i, \tag{3}$$

that enables, in a dynamic perspective, the estimation of time-specific probabilities of belonging to latent class j:

$$\Pr(U_{it}=j)=\sum_{h=1}^{k}\{\Pr(U_{it}=j|U_{i,t-1}=h)\,\Pr(U_{i,t-1}=h)\}, \qquad j=1,\ldots,k, \quad t=2,\ldots,T_i.$$

A peculiarity of our model is that transitions from the status of poverty to the status of non-poverty are not necessarily equally likely across years, unlike commonly formulated in time homogeneous latent Markov models. A similar approach has been adopted in Bartolucci *et al.* (2009). We found this assumption particularly appropriate for our analysis, since for example the probability of transition from non-poverty to poverty status might be potentially larger in years that are characterized by economic crises.

Agents who belong to the status of poverty for the entire period of observation are persistently poor. Given the assumption of local independence, $k=2$, and $U_{it}=2$ indicating the status of poverty, the probability of being persistently poor is

$$\Pr(U_{i1}=\ldots=U_{it}=\ldots=U_{iT_i}=2|Y_i)=\prod_{t=1}^{T_i}\Pr(U_{it}=2|Y_{it}) \tag{4}$$

where $Y_i=(Y_{i1},\ldots,Y_{it},\ldots,Y_{iT_i})$. This definition can be easily modified if persistent poverty is defined differently. Any definition translates to an event which can be computed via the joint posterior distribution of $U_{i1},\ldots,U_{iT_i}$; in general one will obtain sums of products. For example, if any c_i out of T_i years suffice to define a family as persistenly poor, one can recognize the fact that $\Sigma_t U_{it}$ is distributed according to a Poisson–binomial distribution (which is the usual distribution of sums of independent inhomogeneous Bernoulli trials). Let F_l denote the set of all subsets of l integers that can be selected from $\{1,\ldots,T_i\}$. According to the Poisson–binomial distribution it is straightforward to check that

$$\Pr\Big\{\sum_t \mathbb{I}(U_{it}=2)\geqslant c_i\Big\}=\sum_{l=k}^{T_i}\sum_{A\in F_l}\prod_{t\in A}\Pr(U_{it}=2|Y_{it})\prod_{t\notin A}\Pr(U_{it}=1|Y_{it}),$$

where for $c_i=T_i$ we get back equation (4). Similarly, if any spell of c_i consecutive years is needed to define deprivation, then one could define S_l as the set of all subsets of l *consecutive* integers that can be selected from $\{1,\ldots,T_i\}$ and compute

$$\sum_{A\in S_c}\prod_{t\in A}\Pr(U_{it}=2|Y_{it})\prod_{t\notin A}\Pr(U_{it}=1|Y_{it}).$$

3.2. *Estimation strategy*

Estimation of the above model (equations (1)–(3)) requires the assessment of the observed likelihood, which can be expressed as follows:

$$L(\theta)=\prod_{i=1}^{n}\Big\{\sum_{U_{i1}=1}^{k}\sum_{U_{i2}=1}^{k}\ldots\sum_{U_{iT_i}=1}^{k}\Pr(U_{i1})\prod_{t=2}^{T_i}\Pr(U_{it}|U_{i,t-1})\prod_{t=1}^{T_i}\prod_{r=1}^{R}\Pr(Y_{itr}|U_{it})\Big\}^{s_i}, \tag{5}$$

where θ is a shorthand notation for all parameters involved. In equation (5), s_i, $i=1,\ldots,n$, denote the longitudinal sampling weights that are associated with the n households in the panel, which are explicitly incorporated in the model to have estimates that are representative of the entire population. Longitudinal weights are required to mitigate the effect of attrition in panel data. The attrition rates in our panel are 37%, 47% and 55% respectively for Italy, Greece and the UK. The longitudinal weights will give larger weights to the households with characteristics that are similar to those that dropped out of the initial wave. Households that are less likely to leave the sample are instead associated with relatively small weights. The model is estimated by adapting functions that are available in the `LMest` R package (Bartolucci *et al.*, 2015).

Estimation of the parameters through equation (5) is rather cumbersome, since it involves summation over all possible (unknown) configurations. We therefore work with the *complete* log-likelihood

$$l_c(\theta,U)=\sum_{i=1}^{n}s_i\sum_{j=1}^{k}w_{ij1}\log(\pi_j)+\sum_{i=1}^{n}s_i\sum_{t=2}^{T_i}\sum_{j=1}^{k}\sum_{h=1}^{k}z_{ijht}\log(\pi_{jth})$$
$$+\sum_{i=1}^{n}s_i\sum_{t=1}^{T_i}\sum_{r=1}^{R}\sum_{j=1}^{k}w_{ijt}Y_{itr}\log(p_{jr})+\sum_{i=1}^{n}s_i\sum_{t=1}^{T_i}\sum_{r=1}^{R}\sum_{j=1}^{k}w_{ijt}(1-Y_{itr})\log(1-p_{jr}) \qquad (6)$$

where $w_{ijt}=\mathbb{I}(U_{it}=j)$ is the indicator variable of the ith household being in latent state j at time t, and $z_{ijht}=\mathbb{I}(U_{it}=j|U_{i(t-1)}=h)$ is the indicator variable of a transition from state h to state j, for household i, at time t. The complete likelihood (6) can be used to set up an expectation–maximization (EM) algorithm (Dempster *et al.*, 1977), whose details are along the lines of Bartolucci *et al.* (2013), chapter 3.

At the E-step, conditionally on the current parameter values, we compute posterior expected values for the unknown quantities that are involved in equation (6), and the value of the observed likelihood (5) as a by-product. This is done by means of a forward and a backward recursion which are outlined below. At the M-step, conditionally on the expected values above, we update the parameters. The E- and M- steps are alternated until convergence in the likelihood, which can be shown to increase at each step of each iteration.

The forward recursion proceeds sequentially by computing the forward probabilities $f_{it}(j)$, for $i=1,\ldots,n$, as follows.

Step 1: $f_{i1}(j)=\pi_j\Pi_{r=1}^{R}p_{jr}^{Y_{i1r}}(1-p_{jr})^{1-Y_{i1r}}$.
Step 2: if $T_i>1$, for $t\geqslant 2$, $f_{it}(j)=\{\Pi_{r=1}^{R}p_{jr}^{Y_{itr}}(1-p_{jr})^{1-Y_{itr}}\}\Sigma_{h=1}^{k}f_{i,t-1}(h)\pi_{jth}$.

It is straightforward to check that $L(\theta)=\Pi_{i=1}^{n}\{\Sigma_{j=1}^{k}f_{iT_i}(j)\}^{s_i}$. Scaling of the coefficients (Scott, 2002) is the common approach to avoid numerical issues with underflow. In our implementation nonetheless we have preferred to work on the log-scale, with sums also on the log-scale as outlined in the appendix of Farcomeni (2012).

We then proceed with a backward recursion, computing $b_{it}(j)$ as follows.

Step 1: $b_{iT_i}(j)=1$.
Step 2: if $T_i>1$, for $t=T_{i-1},T_{i-2},\ldots,1$,

$$b_{it}(j)=\sum_{h=1}^{k}b_{i,t+1}(h)\pi_{htj}\prod_{r=1}^{R}p_{hr}^{Y_{i,t+1,r}}(1-p_{hr})^{1-Y_{i,t+1,r}}.$$

Once the recursions have been performed, the E-step is concluded by computation of

$$\tilde{w}_{itj}=\frac{f_{it}(j)b_{it}(j)}{\sum_j f_{it}(j)b_{it}(j)},$$

which corresponds to the posterior expected value of w_{itj}, and of

$$\tilde{z}_{ijth}=\frac{\pi_{jth}f_{i,t-1}(h)b_{it}(j)\prod_{r=1}^{R}p_{jr}^{Y_{i,t,r}}(1-p_{jr})^{1-Y_{i,t,r}}}{\sum_j f_{i,t}(j)b_{i,t}(j)}$$

which corresponds to the posterior expected value of z_{ijth}. The two quantities above are plugged into equation (6) and form the basis for the M-step. Indeed, closed form expressions are available for updating all parameters, where the current values of the probabilities can be updated as

$$\hat{\pi}_j = \frac{\sum_{i=1}^{n} s_i \tilde{w}_{i1j}}{\sum_{i=1}^{n} s_i \sum_j \tilde{w}_{i1j}}, \tag{7}$$

$$\hat{\pi}_{jth} = \frac{\sum_{i=1}^{n} s_i \tilde{z}_{ijth}}{\sum_{i=1}^{n} s_i \sum_h \tilde{z}_{ijth}}, \tag{8}$$

and

$$\hat{p}_{jr} = \frac{\sum_{i=1}^{n} s_i \sum_{t=1}^{T_i} Y_{itr} \tilde{w}_{itj}}{\sum_{i=1}^{n} s_i \sum_{t=1}^{T_i} \tilde{w}_{itj}}.$$

At convergence of the algorithm, the quantities above are the maximum likelihood estimates for the parameters and can be used for inference, prediction and interpretation of the results.

A quantity of particular interest, which is a by-product of the EM algorithm, is $\tilde{w}_{itj}$: the posterior probability of household i being in latent state j at time t. This can be used to classify subjects. The quantity $\tilde{w}_{itj}$ summarizes what $Y_{it1}, \ldots, Y_{itR}$ are measuring with error. Similarly, $\hat{p}_{j1}, \ldots, \hat{p}_{jR}$, the item-specific probabilities for each latent class, can be used for assessment of item diagnostic power. Formally, if $j = 1$ corresponds to non-poverty and $j = 2$ to poverty, $1 - \hat{p}_{1r}$ is the *specificity* of the deprivation item (the probability that deprivation item r is equal to 0 for a not-poor agent) and $\hat{p}_{2r}$ the *sensitivity* of the item (the probability that item r is equal to 1 for a poor agent).

An agent is poor if its associated posterior probability $\tilde{w}_{itj}$ is greater than a given threshold. Classification of agents can be done also by using a *continuum* of thresholds instead of setting a fixed arbitrary value. These estimated probabilities depend on a specific combination of presence and absence of deprivation items. Therefore, for any observed agent, the enforced lack of a particular item will lead to an *ex post* probability of being poor that is different from the enforced lack of another item.

3.3. The optimal weighting scheme

The estimated posterior probability of being poor ($j = 2$) in a given year t, $\tilde{w}_{it2} = \Pr(U_{it} = 2 | Y_{it})$, is not a direct scoring system, since we estimate a function

$$\tilde{w}(y) : \{0, 1\}^R \to [0, 1],$$

which associates the probability of being poor with each R-dimensional configuration of 0s and 1s corresponding to absence or presence of deprivation items. Given a list of R items, there are 2^R possible profiles, as illustrated in Table 1.

What we would like to estimate is a one-dimensional scoring system via weights that are associated with each item $\tau_1, \ldots, \tau_R$. A resulting one-dimensional score $S(Y) = \Sigma_{r=1}^{R} \tau_r Y_r$ would be as illustrated in Table 2.

Table 1. Possible configuration of the outcomes and related posterior probabilites

Possible configurations Y	*Posterior probabilities given* Y
$0, 0, \ldots, 0$	$\Pr(U_{it} = 2 \mid 0, 0, \ldots, 0)$
$1, 0, \ldots, 0$	$\Pr(U_{it} = 2 \mid 1, 0, \ldots, 0)$
$0, 1, \ldots, 0$	$\Pr(U_{it} = 2 \mid 0, 1, \ldots, 0)$
$\vdots$	$\vdots$
$1, 1, \ldots, 1$	$\Pr(U_{it} = 2 \mid 1, 1, \ldots, 1)$

Table 2. Possible score values and related posterior probabilities

Possible scores $S(Y)$	*Posterior probabilities given* Y
0	$\Pr(U_{it} = 2 \mid 0, 0, \ldots, 0)$
τ_1	$\Pr(U_{it} = 2 \mid 1, 0, \ldots, 0)$
τ_2	$\Pr(U_{it} = 2 \mid 0, 1, \ldots, 0)$
$\vdots$	$\vdots$
$\Sigma\tau_r$	$\Pr(U_{it} = 2 \mid 1, 1, \ldots, 1)$

Among the possible ways of defining $S(Y)$, we set $\tau_1, \ldots, \tau_R$ such that the following condition is satisfied: for any configuration $Y \in \{0, 1\}^R$ and $Z \in \{0, 1\}^R$ such that $\tilde{w}(Y) > \tilde{w}(Z)$ (i.e. $\Pr(U_{it} = 2 \mid Y) > \Pr(U_{it} = 2 \mid Z)$)

$$\sum_{r=1}^{R} \tau_r Y_r > \sum_{r=1}^{R} \tau_r Z_r, \tag{9}$$

under the constraint that $\Sigma_{r=1}^{R} \tau_r^2 = 1$. The sum-to-1 constraint is imposed for identifiability reasons, and the weights are squared to allow for possible negative values of τ_r when item r is negatively associated with the other items. A combination of $\tau_1, \ldots, \tau_R$ satisfying condition (9) does not necessarily exist. If it does exist, there also exists a one-to-one mapping between $\Sigma\tau_r Y_r$ and $w(Y)$; hence $w(Y)$ and $\Sigma\tau_r Y_r$ contain precisely the same information. To estimate $\tau_1, \ldots, \tau_R$ with the minimal information loss, we now define a different objective function for which a solution always exists. Consider the (ordered) posterior probabilities of being deprived given the configuration Y: $\tilde{w}_{(1)}, \tilde{w}_{(k)}, \ldots, \tilde{w}_{(2^R)}$ and $s_{(k)}$, the sum of the sampling weights that are associated with those households characterized by configuration k, $k = 1, \ldots, 2^R$, in a given year. Let us also define as $S_{(k)}(\tau)$ the kth ordered score given weighting $\tau_1, \ldots, \tau_R$. We want to estimate $\tau_1, \ldots, \tau_R$ minimizing the difference between the ordered posterior probabilities of being deprived $\tilde{w}_{(k)}$ and the corresponding scores $S_{(k)}(\tau_1, \ldots, \tau_R)$.

Our optimization problem can be therefore defined as

$$\inf_{\tau} \sum_{k=1}^{2^R} s_{(k)} \{S_{(k)}(\tau) - \tilde{w}_{(k)}\}^2. \tag{10}$$

If the value at convergence is 0, then it is straightforward to check that condition (9) holds. To solve problem (10) we use a genetic algorithm (Simon, 2013). Our algorithm proceeds from $B-1$ random and one deterministic starting solution (the solution corresponding to an equal weighting scheme). Random solutions are generated uniformly within the parameter space (after logit transformation). The objective function is evaluated for each of the initial solutions. A new iteration then proceeds by generating a new population of B solutions sequentially through elitism, selection, crossover and mutation. Elitism involves retaining the $B\epsilon$ best solutions unchanged. The remaining solutions first undergo a process of selection, i.e. they are discarded with probability proportional to the objective function value. After elitism and selection, crossover occurs with a certain probability. Solutions undergoing crossover are coupled uniformly at random, and new solutions are obtained by convex combination with uniformly random weights (after logit transformation), until $B(1-\epsilon)$ new solutions are available. Finally, new solutions also undergo mutation with a certain probability. When a mutation occurs the solution is perturbed by selecting one dimension uniformly at random and replacing its current value with a value uniformly generated in an interval (in our implementation, between -3 and 3 on the logit scale). Evaluation of the objective function for each of the B new solutions concludes the current iteration. At the final iteration the best solution ever reached is returned as the optimum. In our implementation we use a population size of $B=1000$, $\epsilon=5\%$, a probability of crossover of 80%, a probability of mutation of 5% and 5000 iterations Our implementation is based on functions that are available in the `GA` R package (Scrucca, 2013).

3.4. A general optimal weighting scheme

We generalize the weighting scheme in two directions in this section. First, we consider a general (ordered) outcome Y, rather than simply a binary outcome. Then, we further generalize to the case of an arbitrary k. This follows from a simple generalization of model (1). Let in this section Y_{itr} take possible values from 0 to c_r-1, for a certain $c_r \geqslant 2$. We assume that, for $u>1$,

$$\log\left\{\frac{\Pr(Y_{itr}\geqslant u|U_{it}=j)}{\Pr(Y_{itr}<u|U_{it}=j)}\right\}=\alpha_{jur}, \qquad j=1,\ldots,k. \tag{11}$$

In expression (11) we adopt a general *global* logit parameterization, and there are no constraints on α_{jur}. Two remarks are in order: first, the parameterization is useful for including additional constraints (e.g. that $\alpha_{jur}=\alpha_{jr}+\alpha_{ur}$) and covariates; secondly, in the simplest case we can equivalently parameterize $\Pr(Y_{itr}=u|U_{it}=j)=p_{jur}$, with the constraints $p_{jur}>0$, $\Sigma_u p_{jur}=1$.

First, we proceed by obtaining posterior probabilities $\tilde{w}_2(y_1,\ldots,y_R)=\Pr(U_{it}=2|y_1, y_2,\ldots,y_R)$, for $y_r=0,\ldots,c_r-1$, $r=1,\ldots,R$. To do so, parameters are estimated through an EM algorithm similar to that outlined before. At each E-step of the EM algorithm we compute $p_{jur}=\Pr(Y_{itr}=u|U_{it}=j)$ based on expression (11) and current parameter estimates. The forward and backward recursions are then generalized respectively as follows.

Step 1: $f_{i1}(j)=\pi_j\Pi_{r=1}^R\Pi_{u=1}^{c_r-1}p_{jur}^{I(Y_{i1r}=u)}$.
Step 2: if $T_i>1$, for $t\geqslant 2$, $f_{it}(j)=(\Pi_{r=1}^R\Pi_{u=1}^{c_r-1}p_{jur}^{I(Y_{itr}=u)})\Sigma_{h=1}^k f_{i,t-1}(h)\pi_{jth}$.

And

Step 1: $b_{iT_i}(j)=1$.
Step 2: if $T_i>1$, for $t=T_i-1,T_i-2,\ldots,1$,

$$b_{it}(j)=\sum_{h=1}^k b_{i,t+1}(h)\pi_{htj}\prod_{r=1}^R\prod_{u=1}^{c_r-1}p_{hur}^{I(Y_{i,t+1,r}=u)}.$$

Here $I(\cdot)$ denotes the indicator function. At the M-step, we proceed through equations (7) and (8) and, in general, through a simple Newton–Raphson iteration for updating of α_{jur}-parameters or their reparameterization.

When $k=2$, the scoring system can be defined as $S(Y)=\Sigma_r \tau_r Y_r/c_r$, and optimal weights are found by solving

$$\inf_{\tau} \sum_{j=1}^{\prod_r c_r} s_{(j)} \{S_{(j)}(\tau) - \tilde{w}_2(j)\}^2 \tag{12}$$

similarly to before, where $\tilde{w}_2(j)$ is the jth ordered value of the vector $\tilde{w}_2(y_1,\ldots,y_R)$.

When $k>2$, there are several ways to generalize our problem. Define $\tilde{w}_j(y_1,\ldots,y_R)=\Pr(U_{it}=j|y_1,y_2,\ldots,y_R)$, for $y_r=0,\ldots,c_r-1, r=1,\ldots,R$ and $j=1,\ldots,k$. We indicate here two possible routes. The first proceeds by generally optimizing after ignoring the fact that latent classes might be ordered; hence we can define a different score for each class $S^{(l)}(Y)=\Sigma_r \tau_{lr} Y_r/c_r$ and solve

$$\inf_{\tau_1,\ldots,\tau_{k-1}} \max_{l\geqslant 1} \sum_{j=1}^{\prod_r c_r} s_{(j)} \{S_{(j)}^{(l)}(\tau_l) - \tilde{w}_l(j)\}^2, \tag{13}$$

where $\tilde{w}_l(j)$ is the jth ordered value of the vector $\tilde{w}_l(y_1,\ldots,y_R)$. This will yield k weighting systems τ_l, $l=1,\ldots,k$, each apt at discriminating between the lth and the other classes pooled together. The second can be implemented after ordering latent classes (through estimates of α_{jur}), e.g. from lowest to highest propensity to poverty. Solving

$$\inf_{\tau} \max_{l>1} \sum_{j=1}^{\prod_r c_r} s_{(j)} \left\{ S_{(j)}^{(l)}(\tau_l) - \sum_{u\geqslant l} \tilde{w}_u(j) \right\}^2 \tag{14}$$

gives $k-1$ scores, each optimally discriminating between being in the lth class or above and being in any of the classes that are associated with a lower propensity to poverty (i.e. $U_{it} \geqslant l$ *versus* $U_{jt} < l$).

4. Empirical results

4.1. Data and deprivation items

Material deprivation is analysed in Greece, Italy and the UK: countries whose economic performance widely differed during the period 2010–2013. The reference source for comparative statistics on material deprivation in the European Union (EU) is the Survey on Income and Living Conditions (SILC), in its cross-sectional and longitudinal component. On the consistency of cross-sectional and longitudinal EU SILC income data see, for example Krell *et al.* (2017). For a more general overview of statistical studies using EU SILC data see Longford (2014).

We use the 2013 longitudinal component of the EU SILC (revision 2), released in August 2016. The unit of analysis is the household. The longitudinal component is based on a rotated design with a maximum panel duration of 4 years: a quarter of the original sample is generally replaced each year by new subsample units. We follow households that were successfully interviewed in each of the four survey years 2010–2013, ending up with a four-wave balanced panel. A balanced panel is not necessary for our method, which can be applied directly also to even severely unbalanced panels as soon as dropout is assumed to be ignorable. For generalizations of latent Markov models to non-ignorable drop-out see Bartolucci and Farcomeni (2015) and references therein. A longitudinal weight is assigned to each household in the sample. Since the EU SILC provides longitudinal weights only at personal level, it was necessary to assign a

Table 3. Percentage of households in Greece missing specific items†

Item	*Description*	*Results (%) for 2010*	*Results (%) for 2011*	*Results (%) for 2012*	*Results (%) for 2013*
1	Keep the house warm	24.45	21.74	28.98	28.47
2	1-week holiday	55.62	57.14	55.14	42.75
3	Afford a meal	25.74	9.31	12.34	9.01
4	Unexpected expenses	51.44	40.24	42.06	45.51
5	Telephone	0.66	0.40	0.59	0.59
6	Colour television	0.08	0.08	0.04	0.04
7	Washing machine	1.61	1.74	1.07	0.90
8	Car	7.97	8.59	8.67	7.75
9	Arrears	30.90	33.10	36.73	41.03
	Lack 1 item or more	72.08	66.94	70.07	70.74
	Lack 2 items or more	53.63	49.04	50.19	48.36
	Lack 3 items or more	38.28	31.75	34.90	31.64
	Lack 4 items or more	22.88	16.42	19.00	15.95

†Longitudinal panel 2010–2013, weighted data.

Table 4. Percentage of households in Italy missing specific items†

Item	*Description*	*Results (%) for 2010*	*Results (%) for 2011*	*Results (%) for 2012*	*Results (%) for 2013*
1	Keep the house warm	10.71	18.58	20.91	19.81
2	1-week holiday	38.70	46.34	48.68	48.49
3	Afford a meal	7.55	12.16	16.25	12.95
4	Unexpected expenses	33.26	36.83	41.34	37.48
5	Telephone	1.04	0.22	0.08	0.24
6	Colour television	0.35	0.46	0.14	0.33
7	Washing machine	0.69	0.26	0.27	0.26
8	Car	3.77	3.67	2.19	2.76
9	Arrears	10.28	11.46	11.64	11.15
	Lack 1 item or more	50.11	55.95	57.38	55.42
	Lack 2 items or more	30.54	36.96	40.73	37.49
	Lack 3 items or more	14.75	22.07	24.58	23.15
	Lack 4 items or more	6.76	10.80	14.21	11.92

†Longitudinal panel 2010–2013, weighted data.

weight at household level. All countries in the EU SILC adopt the same probabilistic two-stage sample design but have different weighting sample schemes. In Greece and Italy sampling weights are adjusted to external data sources with calibration to margins to match the corresponding population totals. Since auxiliary variables are collected at both individual and household level, individuals belonging to the same household may have different sampling weights. Therefore, we derived household sampling weights by averaging the sampling weights of its members. In the UK, instead, external variables for calibration are at household level, and therefore individual and household weights are the same, as given by the EU SILC.

In Europe the material deprivation rate is defined by Eurostat (Eurostat, 2012) as the percentage of population living in households that cannot afford at least three of the following $R=9$ attributes or items in a given year (in parentheses, the codes of the variable in the data file):

Table 5. Percentage of households in the UK missing specific items†

Item	*Description*	*Results (%) for 2010*	*Results (%) for 2011*	*Results (%) for 2012*	*Results (%) for 2013*
1	Keep the house warm	8.02	6.96	9.02	8.93
2	1-week holiday	26.61	26.52	27.59	27.45
3	Afford a meal	6.64	5.04	6.69	6.96
4	Unexpected expenses	29.88	32.62	33.85	34.58
5	Telephone	0.00	0.31	0.00	0.00
6	Colour television	0.00	0.16	0.16	0.09
7	Washing machine	0.83	0.61	0.29	0.39
8	Car	6.61	5.34	5.77	5.83
9	Arrears	8.66	8.05	9.17	9.81
	Lack 1 item or more	38.61	40.03	40.87	40.42
	Lack 2 items or more	24.76	25.55	26.72	26.78
	Lack 3 items or more	13.57	12.93	15.29	16.33
	Lack 4 items or more	7.14	5.11	6.82	7.07

†Longitudinal panel 2010–2013, weighted data.

(a) 1, to keep the house adequately warm (HH050);
(b) 2, 1-week annual holiday away from home (HS040);
(c) 3, a meal with meat, chicken and fish or a protein equivalent every second day (HS050);
(d) 4, to face unexpected expenses (HS060);
(e) 5, a telephone (HS070);
(f) 6, a colour television (HS080);
(g) 7, a washing machine (HS100);
(h) 8, a car for private use (HS110);
(i) 9, to avoid arrears on mortgage, rent, utility bills or loans (HS010; HS020; HS030).

Individuals who cannot afford at least four items are considered severely deprived. To identify individuals who are persistently or chronically deprived, Eurostat adopts the 'spell' approach and defines as persistent deprived those Europeans who are deprived in the current year and in at least two out of the preceding three years. For different choices of the set of items and thresholds see Papadopoulos and Tsakloglou (2016).

This nine-item list is fixed for all the EU countries and all the items have the same relevance. The choice of nine deprivation items for defining material deprivation is essentially related to the limited availability of congruous items in the core of the EU SILC. A thematic module on material deprivation was included in the 2009 wave of the EU SILC and Guio *et al.* (2016) proposed to replace the nine-item set with a list of 13 items out of the 33 included in the 2009 thematic wave. The additional items that are needed for calculating a new indicator of *material and social* deprivation have been collected annually in each European country since 2014 (Guio *et al.*, 2017). We could not build an analogous four-wave longitudinal sample containing the new 13 deprivation items. However, results on a single-measurement-occasion latent class model based on the new 13-item scale, which are not reported here, show that qualitatively similar conclusions are obtained with respect to all points at stake.

Although no usage of preference weighting enables a straightforward comparison across countries, the assumption that the same set of goods and services has the same social importance in all countries has been questioned in the literature (Israel and Spannagel, 2013). It should be noted that Guio *et al.* (2017) argued in favour of measurement invariance of the new 13-item score, but under certain parametric assumptions (e.g. that poverty can be summarized by a single Gaussian latent variable). Because of a lack of a methodological framework that is amenable to latent Markov models, we shall not be able to assess measurement invariance formally.

Tables 3–5 report the percentage of households in the panel that cannot afford each single attribute and lack one or more items, two or more, three or more and four or more in Greece, Italy and the UK from 2010 to 2013. In the original data some of the items were collected with three answer categories: have the item; do not have because cannot afford it; do not have for any other reason. These items have been treated as binary variables, coded 1 to indicate the enforced lack (not affordability) of the durable and 0 to indicate the possession of the durable or the lack of the durable for other reasons.

4.2. Sensitivity and specificity of the items

Estimation of the model has been done separately by country, to evaluate country differences that might emerge in the relative importance of each item in the identification of material deprivation; and, for the three countries as a whole, to permit cross-country comparison of material deprivation. The two analyses reflect two ways of interpreting the concept of relative deprivation. The first implicitly defines a deprived household by reference to the standard of material wellbeing of the country where the family resides. The second frames disadvantages in an international (potentially EU-wide) context rather than in national terms, by reference to a single standard for all countries together. As might be expected, a country-specific measure of deprivation might differ from the same measure estimated considering a group of countries as a unique entity, since it is sensitive to gaps in living standards between single nations and potentially produces wide differences in measured levels of disadvantage.

Table 6 reports the association between each item r and the latent categorical variable. For each country, the third column indicates the estimated probability of being poor ($j=2$) in a specific item given that the latent variable assumes the status of poverty, $\hat{p}_{2r}$, and it is a measure of how *sensitive* the item is. The fourth column, instead, indicates the *specificity* of each item r, $1-\hat{p}_{1r}$, i.e. the probability of not lacking item r given that the household is not poor. Ideally,

Table 6. Estimated probability of lacking item r given that the latent state is poverty ($\hat{p}_{2r} \equiv$ sensitivity) and probability of not lacking item r given that the latent state is non-poverty ($1-\hat{p}_{1r} \equiv$ specificity), for Greece, Italy and the UK separately and as a whole: 2010–2013

Item	*Description*	*Results (%) for Greece*		*Results (%) for Italy*		*Results (%) for UK*		*Results (%) for pooled sample*	
		$\hat{p}_{2r}$	$1-\hat{p}_{1r}$	$\hat{p}_{2r}$	$1-\hat{p}_{1r}$	$\hat{p}_{2r}$	$1-\hat{p}_{1r}$	$\hat{p}_{2r}$	$1-\hat{p}_{1r}$
1	Keep the house warm	49.6	92.9	43.4	98.0	21.8	98.1	34.5	98.0
2	1-week holiday	88.9	76.0	92.4	82.4	81.0	95.7	87.4	87.5
3	Afford a meal	31.7	99.0	30.8	98.9	20.9	99.8	25.8	99.5
4	Unexpected expenses	87.3	88.8	83.4	90.3	85.3	91.5	83.5	90.9
5	Telephone	1.2	100.0	0.8	100.0	0.2	100.0	0.7	100.0
6	Colour television	0.1	100.0	0.8	100.0	0.3	100.0	0.5	100.0
7	Washing machine	2.5	99.7	0.9	100.0	1.6	100.0	1.3	100.0
8	Car	15.5	97.6	7.9	99.8	17.9	99.2	12.3	99.5
9	Arrears	58.5	82.9	26.8	98.3	28.7	99.5	29.8	98.5

item r should have sensitivity and specificity equal to 100%: whoever is poor lacks that item and whoever is not poor does not lack that item. It can be seen that generally durable goods (telephone, television and washing machine) are very specific, but not sensitive, attributes. The incapacity to afford a meal and to keep the house adequately warm are also very specific but also quite sensitive. More balanced items, and on the whole more discriminating, are the incapacity of having a 1-week annual holiday away from home and of facing unexpected expenses. Of course, the idea is (optimally) to combine items to obtain a more sensitive and more specific score, as we do below where 'non-informative' items should receive approximately null weights. Formal approaches for selecting discriminant items were adopted in Dean and Raftery (2010) and in Bartolucci *et al.* (2012, 2017).

As shown in Table 6, the discrimination power of the items, in terms of both sensitivity and specificity, differs between countries and among the three countries as a whole. Having arrears, for example, is quite discriminating in Italy and the UK but less in Greece, whereas having a car is much less sensitive in Italy than in Greece and the UK. Durables confirm what has previously been said, even if in Greece having a washing machine is more sensitive than in the other countries.

4.3. Probabilities of being poor and prevalence of poverty

For each household i, the estimated probability of being poor ($j=2$) based on the configuration of vector Y_{it} of the nine items is estimated as $\tilde{w}_{it2}=\Pr(U_{it}=2|Y_{it})$. Within this framework, poverty membership is partially (as only the probability of being poor is determined, and usually it is not 0 or 1) determined by the observed nine-item configuration of each household. Extending the number of latent states from $k=2$ to $k=3$ leaves substantially intact the not-poor group and splits the group of poor people into an extreme povery cluster of small size, and the rest of poor people. Clustering can be obtained by fixing a probability threshold ρ and identifying as poor those households whose posterior probability (of being poor) exceeds the threshold ρ. An optimal thresholding can be defined after having specified a loss function for misclassification. If we give the same weight to misclassification of poor into not poor and of not poor into poor, then the optimal posterior probability threshold ρ is 50%. Similarly, households that are persistently poor or deprived over the period can be identified (see equation (4)).

Table 7 reports the estimated current and persistent rate of deprivation or, more correctly, the current and persistent rate of poverty estimated through manifest deprivations, when $\rho=50\%$.

Comparing Table 7 with Tables 2–5, it is evident that the rates of poverty estimated by using

Table 7. Prevalence of current and persistent poverty in Greece, Italy and the UK and in the pooled sample†

Sample	*Current rate (%)*				*Intertemporal rate (%)*
	2010	*2011*	*2012*	*2013*	
Greece	51.4	46.7	46.9	43.0	23.5
Italy	28.0	32.0	37.0	34.7	12.0
UK	24.3	24.2	25.4	25.7	13.4
Pooled	28.5	31.2	33.7	32.1	19.4

†Households are classified as poor if their estimated posterior probability of belonging to the poverty class is above 50%.

our model are larger than those estimated according to the Eurostat definition for deprivation. This evidence will be further investigated in Section 4.5.

4.4. Mobility

Table 8 reports the transition matrices for the three periods 2010–2011, 2011–2012 and 2012–2013 for Greece, Italy, the UK and the whole sample. The entries of such tables are the estimated probabilities of remaining in the same status, or slipping into or out of poverty between two subsequent years.

As shown in Table 8, mobility in and out of poverty is, on average, larger in Greece and in Italy than in the UK. In Greece the probability of escaping from poverty during the period 2010–2011 is around 23%, whereas in the same period the probability of falling into poverty in Italy is almost 20% and it is larger than the probability of moving out of it (16%). For both countries transitions into poverty are less critical in the later periods. Particularly, in the period 2012–2013 the probabilities of stepping out of poverty are much larger than the probabilities of dropping into it. In the UK, although the estimated probabilities in and out of poverty are the smallest among the three countries, the probability of falling into poverty is always larger than the probability of leaving the status of poverty. Pooling the sample, the estimated proportions of households that are expected to move out of or into poverty are relatively large during the period 2010–2011 (11.5% and 11.3% respectively). Subsequently, the amount of mobility drastically reduces, especially from the status of non-poverty to the status of poverty, conforming to the mobility between periods before the economic recession in Europe. The annual transitions into and out of poverty, estimated in the same way by using the panel data for 2005–2008, are on average about 4% and 9% respectively.

4.5. Dimension reduction

The model that we have estimated can produce an optimal clustering, mapping each of the 2^9

Table 8. Estimated transition matrices in Greece, Italy and the UK and in the pooled sample for the three periods (2010–2011, 2011–2012 and 2012–2013)

Status	*Results for 2010–2011*		*Results for 2011–2012*		*Results for 2012–2013*	
	Not poor	*Poor*	*Not poor*	*Poor*	*Not poor*	*Poor*
Greece						
Not poor	0.930	0.070	0.894	0.106	0.978	0.022
Poor	0.229	0.771	0.106	0.894	0.086	0.914
Italy						
Not poor	0.802	0.198	0.897	0.103	0.940	0.060
Poor	0.159	0.841	0.098	0.902	0.150	0.850
UK						
Not poor	0.910	0.090	0.958	0.042	0.949	0.051
Poor	0.043	0.957	0.033	0.967	0.024	0.976
Pooled sample						
Not poor	0.887	0.113	0.935	0.065	0.964	0.036
Poor	0.115	0.885	0.066	0.934	0.091	0.909

possible configurations of item deprivation to a posterior probability of being poor in a given year, and of persistent poverty (over the entire period). It is nevertheless impractical to work with nine-dimensional vectors. Pennoni and Romeo (2017) discussed how latent Markov models might provide a consistent dimension reduction.

Eurostat works with the sum of this nine-dimensional vector, namely the count of items for each family. In Figs 1(a) and 2(a), 2(b) and 2(c), we argue that this score is a suboptimal summary.

Looking at Fig. 1(a), the case of pooled data, it can be noted that, when the number of items lacking is 3 or more, the probability of being poor is extremely high irrespectively of which items are missing. Therefore, the raw sum of the items, and the threshold fixed at 3, does a good job in identifying the poor. However, there are specific combinations of two lacking items that lead to high probabilities of being poor. The raw sum cannot distinguish such situations from configurations of two lacking items associated with very low probability of being poor. Similarly, there could be configurations of three lacking items that do not necessarily lead to high probability of being poor. This is evident for Greece (see Fig. 2(a)) where specific configurations of three lacking items lead to a probability of belonging to the poverty status of less than 30%. For Italy and especially for the UK, we have a more clear-cut situation where, no matter which are the deprivation items, households with three deprivations and above are almost certain poor. However, suffering from two deprivations may lead to a high probability or to a low probability of being poor depending on which combinations of items households are deprived of. In other words, in our

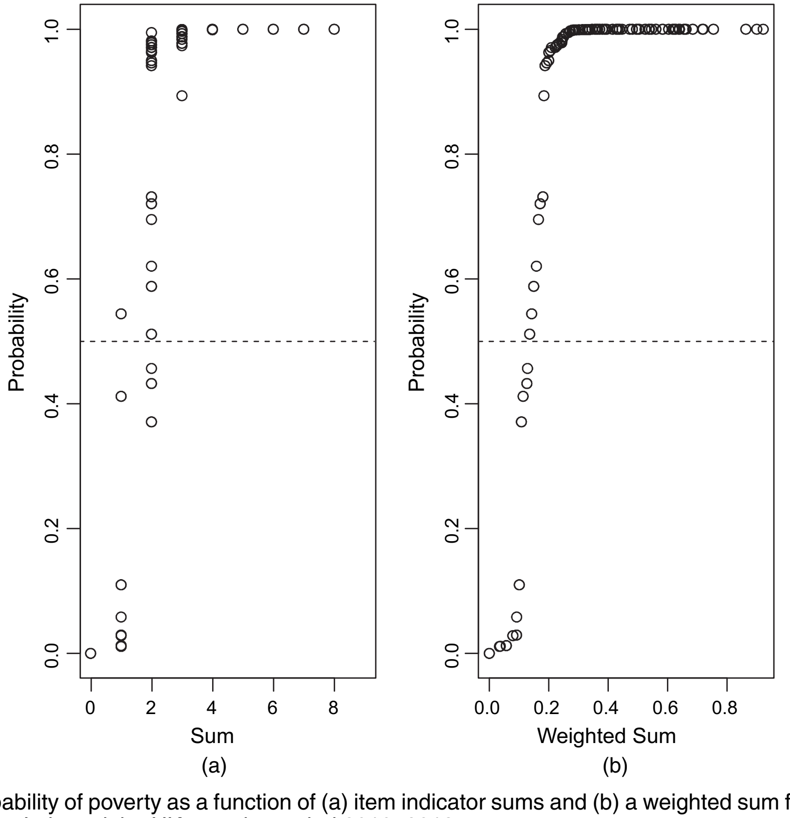

Fig. 1. Probability of poverty as a function of (a) item indicator sums and (b) a weighted sum for the pooled data of Greece, Italy and the UK over the period 2010–2013

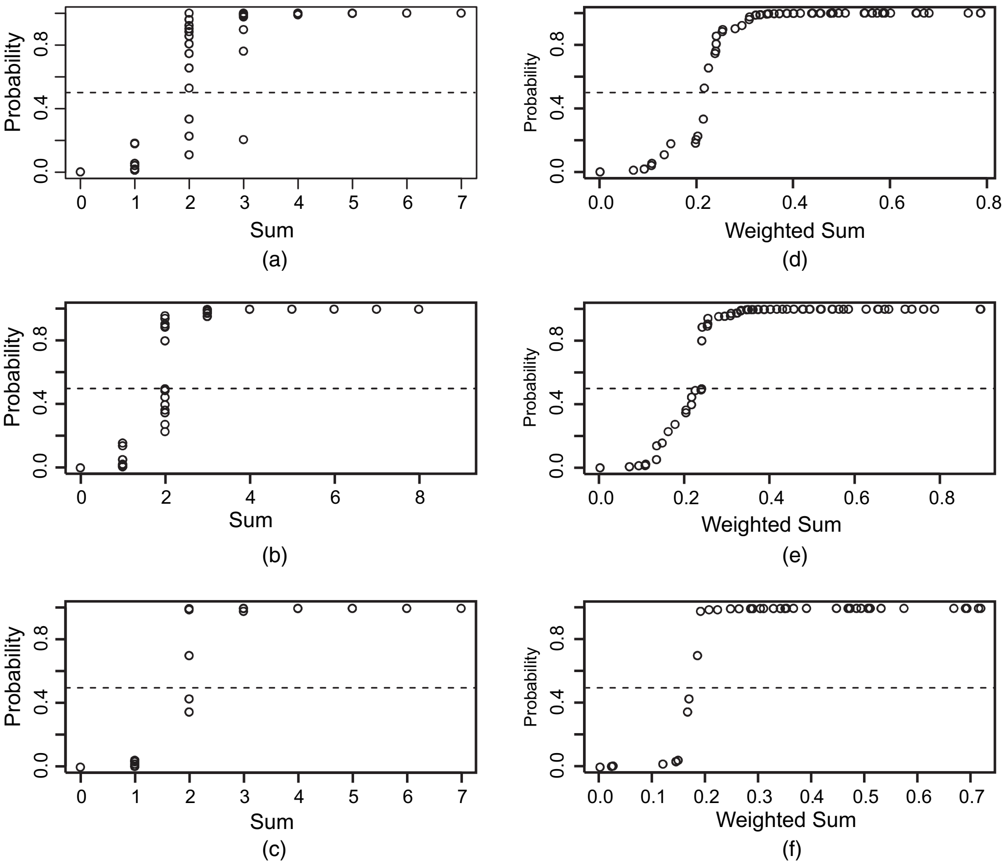

Fig. 2. (a)–(c) Probability of poverty as a function of item indicator sums and (d)–(f) a weighted sum separately for (a), (d) Greece, (b), (e) Italy and (c), (f) the UK over the period 2010–2013

model what matters is not only the number of items lacking but also the whole deprivation profile of the agents. Having said that, it is possible to build up a deprivation score that is coherent with our model by using a weighted deprivation index with weights obtained by solving expression (10). The resulting mapping of scores, obtained through weighted sums, to posterior probabilities are reported in Figs 1(b) and 2(d), 2(e) and 2(f): in correspondence of the 50% probability line there is only one value of the deprivation score above which all households are identified as poor. This is because the probability increases as the weighted deprivation score does.

By inverting the distribution of the optimally weighted sums, we can obtain a pooled threshold for (single-year) deprivation of 0.173, and country-specific thresholds of 0.116, 0.239 and 0.202 for Greece, Italy and the UK respectively. These are the smallest values corresponding to $\Pr(U_{it}=2|\Sigma_{r=1}^{R}\tau_r Y_r)>50\%$ for each of the four data sets. For persistent poverty (defined as $T=4$ consecutive years in the latent status of deprived), the thresholds are 0.221 for pooled data, and 0.139, 0.240 and 0.214 for Greece, Italy and the UK respectively. Note that these thresholds imply that, if a family interviewed for a *single* year has $\Sigma_{r=1}^{R}\tau_r Y_r > \text{th}$, where th is the opportune threshold, then we can predict that the family is persistently poor (namely that it is experiencing a spell of at least T consecutive years in the deprivation status).

We have two additional notes. First, the null hypothesis that weights are equal is rejected for each of the four data configurations. Hence treating the deprivation items equally and simply counting the number of items lacking produces, according to our model, classifications that do not completely agree with estimated posterior probabilities. Second, the null hypothesis that weights are equal across countries is also rejected, implying that poverty is very likely to be context dependent at least when considering these countries. The test has been performed as a classical permutation test. To test whether the weights were equal across countries, for instance, weights were randomly shuffled among for corresponding items among different countries and the weighted sum computed with shuffled weights. Then we computed the upper quantile of the total weighted sum with unshuffled weights based on the empirical permutation distribution obtained in this way.

To compare our methodology with the counting approach, we estimated the percentages of households that are identified as poor or not poor by the weighted score (with thresholding corresponding to a probability line of 50%) and by unweighted score (with thresholding fixed at $\lambda = 3$, as Eurostat does). Table 9 reports these cross-classification tables for the pooled data and by country for the year 2013.

As previously pointed out, the prevalence rates that are estimated in our model are consistently higher than those estimated by the raw sum of deprivations. Generally, who is identified as poor or deprived by the unweighted score method is also identified as poor by the weighted score method. The only exception is for Greece in 2010 where a small fraction of households are identified as not poor by the weighted score and poor by the unweighted score. The weighted score method identifies as poor an additional group of households. These households are characterized, as said above, by specific configurations of two lacking items that lead to a weighted index above the threshold. It is interesting to check whether this additional group (which can be labelled 'P–NP') has characteristics that are more similar to the group that is identified as poor by both methods (labelled 'P–P') or to the group that is identified as not poor in both methods (labelled 'NP–NP'). We therefore selected a set of independent variables that are known to be correlated with deprivation (Guio *et al.*, 2016): equivalent household disposable income; self-judgement about the ability to make ends meet; self-perceived health status. The association between these variables and the posterior probabilities of being poor, w_{it2}, has been confirmed also for our data by estimating a log-ratio-transformed regression model. Table 10 reports the average income of the households identified as P–P, P–NP and NP–NP by country and for the pooled data for the year 2013. Similar results have been obtained for the other years. Income of the additional poor group (P–NP) is always higher than the average income of the P–P group

Table 9. Percentage of households identified as poor and not poor according to the weighted and unweighted sum of deprivations†

Status	*Unweighted sum of deprivations*											
	Greece			*Italy*			*UK*			*Pooled*		
	Poor (%)	*Not poor (%)*	*Total (%)*	*Poor (%)*	*Not poor (%)*	*Total (%)*	*Poor (%)*	*Not poor (%)*	*Total (%)*	*Poor (%)*	*Not poor (%)*	*Total (%)*
Weighted sum of deprivations												
Poor	31.6	11.4	43.0	23.2	11.2	34.4	16.3	9.4	25.7	20.2	11.9	32.1
Not poor	0.0	57.0	57.0	0.0	65.6	65.6	0.0	74.3	74.3	0.0	67.9	67.9
Total	31.6	68.4	100.0	23.2	76.8	100.0	16.3	83.7	100.0	20.2	79.8	100.0

†Percentages estimated by country and for the pooled data, year 2013.

Table 10. Equivalized disposable income by subgroups in Greece, Italy and the UK and in the pooled data, year 2013

Status	*Unweighted sum of deprivations*							
	Greece		*Italy*		*UK*		*Pooled*	
	Poor (€)	*Not poor (€)*	*Poor (€)*	*Not poor (€)*	*Poor (€)*	*Not poor (€)*	*Poor (€)*	*Not poor (€)*
Weighted sum								
Poor	6031	7242	12335	14912	14674	16108	10627	13425
Not poor	—	11189	—	21933	—	26346	—	21254

Table 11. Percentage of households that declare difficulties in making ends meet by subgroups in Greece, Italy and the UK and in the pooled data, year 2013

Status	*Unweighted sum of deprivations*							
	Greece		*Italy*		*UK*		*Pooled*	
	Poor (%)	*Not poor (%)*	*Poor (%)*	*Not poor (%)*	*Poor (%)*	*Not poor (%)*	*Poor (%)*	*Not poor (%)*
Weighted sum								
Poor	94.4	79.5	79.0	63.0	55.9	37.1	80.0	60.0
Not poor	—	66.1	—	19.1	—	4.6	—	23.5

Table 12. Percentage of households with at least one member in bad health conditions by subgroups in Greece, Italy and the UK and in the pooled data, year 2013

Status	*Unweighted sum of deprivations*							
	Greece		*Italy*		*UK*		*Pooled*	
	Poor (%)	*Not poor (%)*	*Poor (%)*	*Not poor (%)*	*Poor (%)*	*Not poor (%)*	*Poor (%)*	*Not poor (%)*
Weighted sum								
Poor	26.3	26.7	31.9	28.8	28.0	19.8	29.3	25.5
Not poor	—	20.2	—	17.3	—	7.4	—	15.0

(around 20% higher for Greece and Italy and around 10% higher in the UK), but much lower than the average income of the NP–NP group (around 55%, 50% and 60% lower for Greece, Italy and the UK respectively). Also, in terms of economic strain, the percentage of households that declare difficulties in making ends meet are, for the pooled data, 80% in the P–P group, 60% in the NP–P group and 23.5% in the NP–NP group. Similar patterns are present in the single countries (Table 11). Finally, in terms of health problems (Table 12) the proportions of households that have at least one member in bad health conditions are close or very close between the P–P and the P–NP group and much lower in the NP–NP group. This evidence seems to corroborate the presence of an additional group of poor families, leading to higher rates of prevalence of poverty. This group corresponds to families that are detected by our modelling framework but not by the commonly used counting approach. Similarly, the larger rates of poverty that we found seem to be due to the ability of our model to discriminate households with the same number of items lacking in poor and not poor, thus identifying poor families with fewer than three items lacking.

5. Concluding remarks

The measurement of poverty is challenging and involves both methodological and substantive issues. Adopting a direct approach of measurement, i.e. through a set of deprivations, the most relevant are definition of an appropriate list of deprivation items, along with their relative importance, identification of a deprivation score for each individual or household, definition of a proper threshold for classifying who is poor and who is not, and assessing transitions from and to the poverty status. A technical problem that also arises is how to deal with categorical variables that are the typical form of deprivation items. We proposed a latent Markov model for categorical longitudinal data that can solve some of the measurement issues. Within this framework, we treat the real status of poverty as a latent variable, whereas the observable characteristics in the list of items represent deprivations with measurement error. Households of three EU countries (Greece, Italy and the UK) were clustered in poor and not poor in terms of both current and persistent poverty, based on the posterior probability of being in the latent class of poverty. Chronically poor households have been identified looking at the joint distribution of latent poverty status for all years under consideration.

We estimated four models: one for each country separately and one pooling the data. The first three models lead to parameter estimates that are exactly equal to those that would have been obtained after pooling the data and including two country dummies to parameterize both the manifest and the latent distributions. Including dummies only for the latent or only for the manifest distribution would lead to different parameter estimates that, in the context of poverty analysis, would be somewhat difficult to interpret.

The methodological approach that we proposed is made by two independent components: a latent transition model allowing agents, even temporarily, to change their latent status, and a weighting scheme to avoid the direct use of multi-dimensional configurations for identification of poor people. We obtained strong evidence that latent transitions occur. Whereas the first period of the analysis (2010–2011) reveals substantial movements into and out of poverty, in the next period (2011–2012) we observe a higher level of persistence in the status of poverty and non-poverty, with a probability of escaping from poverty similar to the probability of dropping into it. Only in the third period (2011–2012) are the transition probability matrices similar to those referring to periods before the economic recession.

Our approach has mostly a diagnostic purpose. Policy indications might be better obtained by

(a) evaluating, as outcomes, also measures of work intensity, disposable income, etc. and
(b) possibly increasing the number of latent states.

A promising approach is also given by letting the number of latent states change over time, as in Anderson *et al*. (2017).

Optimal weights of some deprivation items differ significantly by country: having arrears is quite discriminating in Italy and the UK but not in Greece, whereas having a car is much less discriminating in Italy than in Greece and the UK. Durables are in general not discriminant, even if in Greece having a washing machine is more sensitive than in the other countries. We note, however, that this is not a formal assessment of measurement invariance, as methods for evaluating this concept are not yet available under our latent Markov framework. This is ground for further methodological development.

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