

Supplementary Material for “Directional quantile classifiers”

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Abstract

We introduce classifiers based on directional quantiles. We derive theoretical results for selecting optimal quantile levels given a direction, and, conversely, an optimal direction given a quantile level. We also show that the misclassification rate is infinitesimal if population distributions differ by at most a location shift and if the number of directions is allowed to diverge at the same rate of the problem’s dimension. We illustrate the satisfactory performance of our proposed classifiers in both small and high dimensional settings via a simulation study and a real data example. The code implementing the proposed methods is publicly available in the R package `Qtools`.

Keywords: L_1 distance, Supervised Classification, Quantiles for multivariate data.

1 Additional simulation results

In Tables 1–4 we show simulation results for the settings described in the main paper.

Table 1: Misclassification rates averaged over 100 replications with standard errors in brackets for ten classifiers (DQC, directional quantile classifier; Centroid, centroid classifier; Median, median classifier; CQC, componentwise quantile classifier; EQC, ensemble quantile classifier; LDA, linear discriminant analysis; KNN, k-nearest neighbour; PLR, penalised logistic regression; SVM, support vector machines; Bayes, naïve Bayes) in the second scenario where populations have Student’s t_3 marginals.

Dimension p	Uncorrelated				Correlated			
	10	50	100	500	10	50	100	500
<i>Sample size $n = 50$</i>								
DQC	0.374 (0.075)	0.238 (0.062)	0.156 (0.057)	0.012 (0.016)	0.305 (0.073)	0.090 (0.047)	0.022 (0.021)	0.000 (0.000)
Centroid	0.380 (0.071)	0.268 (0.062)	0.195 (0.055)	0.031 (0.025)	0.326 (0.069)	0.084 (0.046)	0.010 (0.015)	0.000 (0.000)
Median	0.356 (0.071)	0.225 (0.058)	0.142 (0.050)	0.009 (0.013)	0.329 (0.061)	0.089 (0.041)	0.016 (0.020)	0.000 (0.000)
CQC	0.382 (0.069)	0.279 (0.074)	0.262 (0.105)	0.364 (0.073)	0.348 (0.069)	0.158 (0.092)	0.175 (0.076)	0.001 (0.004)
EQC	0.375 (0.080)	0.235 (0.055)	0.151 (0.050)	0.012 (0.017)	0.337 (0.062)	0.098 (0.045)	0.022 (0.025)	0.000 (0.000)
LDA	0.396 (0.079)	0.436 (0.079)	0.313 (0.067)	0.284 (0.081)	0.306 (0.074)	0.336 (0.102)	0.148 (0.049)	0.044 (0.043)
KNN	0.416 (0.073)	0.359 (0.060)	0.300 (0.056)	0.183 (0.060)	0.375 (0.075)	0.155 (0.061)	0.040 (0.035)	0.000 (0.000)
PLR	0.394 (0.080)	0.295 (0.066)	0.210 (0.066)	0.034 (0.029)	0.310 (0.071)	0.083 (0.046)	0.011 (0.015)	0.000 (0.000)
SVM	0.402 (0.089)	0.292 (0.066)	0.215 (0.068)	0.028 (0.026)	0.313 (0.073)	0.085 (0.047)	0.014 (0.017)	0.000 (0.000)
Bayes	0.421 (0.077)	0.387 (0.077)	0.336 (0.067)	0.218 (0.059)	0.383 (0.067)	0.215 (0.062)	0.101 (0.048)	0.000 (0.002)
<i>Sample size $n = 100$</i>								
DQC	0.352 (0.048)	0.207 (0.042)	0.122 (0.037)	0.006 (0.008)	0.286 (0.048)	0.079 (0.031)	0.018 (0.013)	0.000 (0.000)
Centroid	0.361 (0.051)	0.235 (0.044)	0.163 (0.040)	0.015 (0.010)	0.304 (0.051)	0.061 (0.026)	0.005 (0.007)	0.000 (0.000)
Median	0.350 (0.050)	0.179 (0.036)	0.099 (0.033)	0.002 (0.005)	0.301 (0.046)	0.066 (0.024)	0.008 (0.009)	0.000 (0.000)
CQC	0.368 (0.056)	0.199 (0.053)	0.126 (0.051)	0.340 (0.085)	0.307 (0.049)	0.083 (0.033)	0.048 (0.045)	0.002 (0.004)
EQC	0.359 (0.054)	0.194 (0.042)	0.107 (0.033)	0.006 (0.009)	0.298 (0.049)	0.079 (0.030)	0.014 (0.011)	0.000 (0.000)
LDA	0.368 (0.054)	0.309 (0.053)	0.422 (0.072)	0.197 (0.048)	0.271 (0.043)	0.083 (0.034)	0.291 (0.089)	0.006 (0.008)
KNN	0.400 (0.047)	0.349 (0.057)	0.288 (0.053)	0.119 (0.061)	0.342 (0.049)	0.120 (0.045)	0.026 (0.022)	0.000 (0.000)
PLR	0.369 (0.054)	0.273 (0.052)	0.189 (0.038)	0.018 (0.012)	0.274 (0.045)	0.063 (0.027)	0.006 (0.008)	0.000 (0.000)
SVM	0.372 (0.053)	0.265 (0.049)	0.175 (0.039)	0.017 (0.011)	0.289 (0.047)	0.063 (0.026)	0.009 (0.010)	0.000 (0.000)
Bayes	0.414 (0.054)	0.342 (0.050)	0.300 (0.045)	0.192 (0.039)	0.356 (0.048)	0.168 (0.039)	0.076 (0.025)	0.000 (0.000)
<i>Sample size $n = 500$</i>								
DQC	0.334 (0.024)	0.193 (0.018)	0.113 (0.014)	0.004 (0.003)	0.276 (0.020)	0.077 (0.012)	0.018 (0.006)	0.000 (0.000)
Centroid	0.342 (0.022)	0.206 (0.020)	0.128 (0.014)	0.007 (0.003)	0.275 (0.021)	0.046 (0.010)	0.003 (0.002)	0.000 (0.000)
Median	0.322 (0.020)	0.150 (0.016)	0.073 (0.011)	0.001 (0.001)	0.278 (0.020)	0.051 (0.010)	0.005 (0.003)	0.000 (0.000)
CQC	0.330 (0.024)	0.159 (0.018)	0.079 (0.013)	0.008 (0.007)	0.284 (0.020)	0.054 (0.010)	0.007 (0.005)	0.000 (0.001)
EQC	0.327 (0.021)	0.157 (0.019)	0.076 (0.011)	0.001 (0.001)	0.275 (0.021)	0.052 (0.009)	0.007 (0.005)	0.000 (0.000)
LDA	0.344 (0.021)	0.216 (0.020)	0.153 (0.015)	0.417 (0.041)	0.256 (0.020)	0.029 (0.007)	0.002 (0.002)	0.166 (0.080)
KNN	0.388 (0.026)	0.320 (0.023)	0.227 (0.022)	0.083 (0.027)	0.308 (0.021)	0.093 (0.014)	0.011 (0.006)	0.000 (0.000)
PLR	0.344 (0.021)	0.218 (0.021)	0.158 (0.015)	0.011 (0.004)	0.257 (0.020)	0.041 (0.010)	0.003 (0.003)	0.000 (0.000)
SVM	0.350 (0.023)	0.231 (0.022)	0.132 (0.015)	0.010 (0.004)	0.258 (0.019)	0.038 (0.010)	0.005 (0.003)	0.000 (0.000)
Bayes	0.359 (0.024)	0.274 (0.023)	0.238 (0.020)	0.142 (0.018)	0.306 (0.023)	0.108 (0.016)	0.042 (0.010)	0.000 (0.000)

Table 2: Misclassification rates averaged over 100 replications with standard errors in brackets for ten classifiers (DQC, directional quantile classifier; Centroid, centroid classifier; Median, median classifier; CQC, componentwise quantile classifier; EQC, ensemble quantile classifier; LDA, linear discriminant analysis; KNN, k-nearest neighbour; PLR, penalised logistic regression; SVM, support vector machines; Bayes, naïve Bayes) in the third scenario where populations have $\log|t_3|$ marginals.

Dimension p	Uncorrelated					Correlated				
	10	50	100	500	500	10	50	100	500	
<i>Sample size $n = 50$</i>										
DQC	0.319 (0.070)	0.155 (0.049)	0.078 (0.037)	0.001 (0.006)	0.322 (0.075)	0.142 (0.056)	0.065 (0.039)	0.001 (0.003)		
Centroid	0.329 (0.077)	0.179 (0.052)	0.091 (0.035)	0.002 (0.008)	0.334 (0.070)	0.163 (0.056)	0.089 (0.042)	0.001 (0.004)		
Median	0.343 (0.078)	0.197 (0.063)	0.100 (0.041)	0.003 (0.008)	0.330 (0.071)	0.178 (0.056)	0.096 (0.042)	0.002 (0.007)		
CQC	0.349 (0.072)	0.219 (0.078)	0.206 (0.085)	0.112 (0.050)	0.340 (0.068)	0.209 (0.075)	0.193 (0.083)	0.058 (0.035)		
EQC	0.343 (0.075)	0.193 (0.062)	0.098 (0.046)	0.002 (0.006)	0.334 (0.074)	0.170 (0.063)	0.087 (0.039)	0.001 (0.004)		
LDA	0.348 (0.081)	0.405 (0.089)	0.255 (0.079)	0.208 (0.067)	0.347 (0.076)	0.403 (0.094)	0.248 (0.066)	0.179 (0.070)		
KNN	0.378 (0.072)	0.263 (0.055)	0.197 (0.056)	0.054 (0.040)	0.378 (0.078)	0.257 (0.073)	0.273 (0.073)	0.017 (0.024)		
PLR	0.342 (0.074)	0.203 (0.056)	0.116 (0.052)	0.003 (0.009)	0.339 (0.074)	0.187 (0.059)	0.100 (0.046)	0.001 (0.003)		
SVM	0.340 (0.081)	0.181 (0.050)	0.107 (0.048)	0.002 (0.007)	0.359 (0.079)	0.176 (0.053)	0.114 (0.048)	0.001 (0.003)		
Bayes	0.373 (0.074)	0.271 (0.065)	0.206 (0.055)	0.046 (0.028)	0.365 (0.071)	0.256 (0.069)	0.185 (0.056)	0.031 (0.023)		
<i>Sample size $n = 100$</i>										
DQC	0.308 (0.052)	0.130 (0.033)	0.056 (0.027)	0.000 (0.002)	0.303 (0.053)	0.119 (0.033)	0.053 (0.024)	0.000 (0.002)		
Centroid	0.320 (0.052)	0.151 (0.035)	0.074 (0.027)	0.001 (0.003)	0.315 (0.056)	0.141 (0.036)	0.072 (0.028)	0.000 (0.002)		
Median	0.329 (0.052)	0.159 (0.037)	0.077 (0.028)	0.001 (0.003)	0.326 (0.045)	0.149 (0.036)	0.072 (0.030)	0.001 (0.003)		
CQC	0.334 (0.053)	0.158 (0.047)	0.083 (0.038)	0.088 (0.030)	0.318 (0.059)	0.135 (0.045)	0.074 (0.044)	0.024 (0.016)		
EQC	0.323 (0.046)	0.155 (0.039)	0.064 (0.025)	0.000 (0.002)	0.308 (0.052)	0.132 (0.044)	0.059 (0.025)	0.000 (0.000)		
LDA	0.329 (0.057)	0.239 (0.052)	0.381 (0.064)	0.105 (0.038)	0.320 (0.055)	0.219 (0.051)	0.380 (0.074)	0.099 (0.033)		
KNN	0.392 (0.056)	0.227 (0.042)	0.174 (0.043)	0.026 (0.019)	0.413 (0.054)	0.249 (0.051)	0.153 (0.043)	0.016 (0.017)		
PLR	0.327 (0.056)	0.189 (0.043)	0.093 (0.031)	0.001 (0.004)	0.319 (0.052)	0.171 (0.041)	0.090 (0.030)	0.001 (0.002)		
SVM	0.345 (0.053)	0.160 (0.033)	0.075 (0.027)	0.001 (0.004)	0.325 (0.056)	0.162 (0.038)	0.082 (0.028)	0.000 (0.002)		
Bayes	0.349 (0.056)	0.219 (0.043)	0.145 (0.037)	0.013 (0.011)	0.342 (0.056)	0.199 (0.040)	0.135 (0.035)	0.008 (0.010)		
<i>Sample size $n = 500$</i>										
DQC	0.293 (0.020)	0.120 (0.014)	0.050 (0.009)	0.000 (0.000)	0.295 (0.020)	0.113 (0.013)	0.044 (0.010)	0.000 (0.000)		
Centroid	0.299 (0.018)	0.129 (0.014)	0.055 (0.009)	0.000 (0.001)	0.299 (0.019)	0.121 (0.014)	0.050 (0.011)	0.000 (0.000)		
Median	0.306 (0.020)	0.129 (0.017)	0.053 (0.010)	0.000 (0.001)	0.303 (0.020)	0.122 (0.015)	0.048 (0.009)	0.000 (0.001)		
CQC	0.291 (0.023)	0.112 (0.015)	0.042 (0.011)	0.004 (0.004)	0.283 (0.022)	0.092 (0.013)	0.028 (0.008)	0.003 (0.003)		
EQC	0.293 (0.021)	0.111 (0.015)	0.041 (0.011)	0.000 (0.000)	0.284 (0.021)	0.093 (0.014)	0.029 (0.007)	0.000 (0.000)		
LDA	0.299 (0.018)	0.140 (0.014)	0.075 (0.013)	0.378 (0.055)	0.300 (0.020)	0.131 (0.015)	0.067 (0.013)	0.385 (0.048)		
KNN	0.333 (0.024)	0.222 (0.021)	0.121 (0.017)	0.009 (0.005)	0.335 (0.022)	0.208 (0.019)	0.109 (0.016)	0.006 (0.004)		
PLR	0.300 (0.019)	0.145 (0.014)	0.080 (0.012)	0.000 (0.001)	0.301 (0.021)	0.135 (0.015)	0.071 (0.013)	0.000 (0.000)		
SVM	0.299 (0.018)	0.130 (0.013)	0.058 (0.010)	0.000 (0.001)	0.285 (0.020)	0.123 (0.014)	0.053 (0.011)	0.000 (0.000)		
Bayes	0.301 (0.022)	0.141 (0.015)	0.068 (0.012)	0.002 (0.002)	0.289 (0.019)	0.116 (0.014)	0.051 (0.010)	0.001 (0.001)		

Table 3: Misclassification rates averaged over 100 replications with standard errors in brackets for ten classifiers (DQC, directional quantile classifier; Centroid, centroid classifier; Median, median classifier; CQC, componentwise quantile classifier; EQC, ensemble quantile classifier; LDA, linear discriminant analysis; KNN, k-nearest neighbour; PLR, penalised logistic regression; SVM, support vector machines; Bayes, naïve Bayes) in the fourth scenario where populations have $\log |t_3|$ marginals with opposite skewness.

Dimension p	Uncorrelated					Correlated				
	10	50	100	500	500	10	50	100	500	500
<i>Sample size $n = 50$</i>										
DQC	0.120 (0.046)	0.003 (0.007)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)	0.048 (0.031)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)
Centroid	0.129 (0.041)	0.004 (0.008)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)	0.050 (0.031)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)
Median	0.252 (0.055)	0.065 (0.037)	0.020 (0.020)	0.000 (0.000)	0.000 (0.000)	0.105 (0.051)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)
CQC	0.206 (0.062)	0.027 (0.023)	0.003 (0.008)	0.000 (0.000)	0.000 (0.000)	0.097 (0.045)	0.000 (0.002)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)
EQC	0.238 (0.060)	0.053 (0.030)	0.011 (0.015)	0.000 (0.000)	0.000 (0.000)	0.102 (0.049)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)
LDA	0.153 (0.053)	0.264 (0.086)	0.033 (0.027)	0.010 (0.015)	0.000 (0.000)	0.068 (0.041)	0.160 (0.103)	0.000 (0.003)	0.000 (0.000)	0.000 (0.000)
KNN	0.194 (0.058)	0.026 (0.025)	0.002 (0.008)	0.000 (0.000)	0.000 (0.000)	0.068 (0.039)	0.001 (0.003)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)
PLR	0.167 (0.055)	0.006 (0.010)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)	0.066 (0.040)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)
SVM	0.134 (0.043)	0.005 (0.009)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)	0.052 (0.030)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)
Bayes	0.181 (0.058)	0.022 (0.024)	0.005 (0.011)	0.000 (0.000)	0.000 (0.000)	0.067 (0.041)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)
<i>Sample size $n = 100$</i>										
DQC	0.114 (0.032)	0.003 (0.006)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)	0.040 (0.019)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)
Centroid	0.119 (0.034)	0.003 (0.005)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)	0.037 (0.017)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)
Median	0.232 (0.043)	0.052 (0.024)	0.015 (0.013)	0.000 (0.000)	0.000 (0.000)	0.090 (0.029)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)
CQC	0.196 (0.041)	0.029 (0.020)	0.002 (0.005)	0.000 (0.000)	0.000 (0.000)	0.081 (0.029)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)
EQC	0.220 (0.040)	0.046 (0.024)	0.008 (0.009)	0.000 (0.000)	0.000 (0.000)	0.087 (0.028)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)
LDA	0.126 (0.036)	0.031 (0.019)	0.254 (0.087)	0.000 (0.002)	0.000 (0.002)	0.044 (0.021)	0.000 (0.002)	0.135 (0.096)	0.000 (0.000)	0.000 (0.000)
KNN	0.168 (0.043)	0.025 (0.015)	0.002 (0.005)	0.000 (0.000)	0.000 (0.000)	0.059 (0.024)	0.000 (0.001)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)
PLR	0.123 (0.032)	0.005 (0.008)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)	0.053 (0.027)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)
SVM	0.121 (0.033)	0.003 (0.005)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)	0.038 (0.019)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)
Bayes	0.134 (0.036)	0.009 (0.009)	0.000 (0.002)	0.000 (0.000)	0.000 (0.000)	0.044 (0.020)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)
<i>Sample size $n = 500$</i>										
DQC	0.113 (0.015)	0.003 (0.002)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)	0.040 (0.009)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)
Centroid	0.115 (0.015)	0.003 (0.002)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)	0.038 (0.008)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)
Median	0.228 (0.019)	0.048 (0.010)	0.009 (0.004)	0.000 (0.000)	0.000 (0.000)	0.089 (0.013)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)
CQC	0.184 (0.020)	0.017 (0.006)	0.002 (0.002)	0.000 (0.000)	0.000 (0.000)	0.085 (0.011)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)
EQC	0.212 (0.019)	0.035 (0.009)	0.006 (0.003)	0.000 (0.000)	0.000 (0.000)	0.086 (0.012)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)
LDA	0.115 (0.014)	0.004 (0.003)	0.000 (0.001)	0.249 (0.072)	0.000 (0.000)	0.040 (0.008)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)	0.091 (0.081)
KNN	0.134 (0.016)	0.011 (0.005)	0.001 (0.002)	0.000 (0.000)	0.000 (0.000)	0.047 (0.009)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)
PLR	0.116 (0.014)	0.004 (0.003)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)	0.041 (0.008)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)
SVM	0.114 (0.014)	0.003 (0.002)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)	0.039 (0.008)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)
Bayes	0.108 (0.014)	0.003 (0.002)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)	0.037 (0.008)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)

Table 4: Misclassification rates averaged over 100 replications with standard errors in brackets for ten classifiers (DQC, directional quantile classifier; Centroid, centroid classifier; Median, median classifier; CQC, componentwise quantile classifier; EQC, ensemble quantile classifier; LDA, linear discriminant analysis; KNN, k-nearest neighbour; PLR, penalised logistic regression; SVM, support vector machines; Bayes, naïve Bayes) in the fifth scenario where populations have exponential marginals.

Dimension p	Uncorrelated			Correlated		
	10	50	100	100	50	100
<i>Sample size $n = 50$</i>						
DQC	0.252 (0.071)	0.084 (0.043)	0.023 (0.019)	0.000 (0.000)	0.197 (0.060)	0.001 (0.004)
Centroid	0.264 (0.068)	0.110 (0.046)	0.039 (0.026)	0.000 (0.003)	0.223 (0.065)	0.000 (0.003)
Median	0.259 (0.064)	0.088 (0.044)	0.030 (0.026)	0.000 (0.000)	0.232 (0.063)	0.001 (0.003)
CQC	0.122 (0.063)	0.014 (0.019)	0.002 (0.007)	0.000 (0.000)	0.124 (0.062)	0.001 (0.005)
EQC	0.168 (0.058)	0.013 (0.017)	0.001 (0.004)	0.000 (0.000)	0.152 (0.059)	0.000 (0.000)
LDA	0.270 (0.069)	0.372 (0.097)	0.185 (0.066)	0.135 (0.056)	0.215 (0.069)	0.060 (0.041)
KNN	0.307 (0.066)	0.192 (0.066)	0.119 (0.053)	0.006 (0.011)	0.255 (0.059)	0.005 (0.011)
PLR	0.267 (0.067)	0.137 (0.054)	0.050 (0.033)	0.000 (0.003)	0.224 (0.060)	0.001 (0.005)
SVM	0.261 (0.068)	0.118 (0.048)	0.048 (0.033)	0.000 (0.002)	0.236 (0.060)	0.001 (0.004)
Bayes	0.236 (0.071)	0.113 (0.050)	0.064 (0.040)	0.002 (0.006)	0.213 (0.062)	0.008 (0.013)
<i>Sample size $n = 100$</i>						
DQC	0.253 (0.053)	0.077 (0.030)	0.021 (0.013)	0.000 (0.000)	0.194 (0.037)	0.001 (0.003)
Centroid	0.259 (0.051)	0.092 (0.030)	0.030 (0.014)	0.000 (0.001)	0.206 (0.041)	0.000 (0.002)
Median	0.262 (0.052)	0.073 (0.026)	0.023 (0.016)	0.000 (0.000)	0.213 (0.047)	0.000 (0.000)
CQC	0.084 (0.034)	0.003 (0.006)	0.000 (0.001)	0.000 (0.000)	0.090 (0.043)	0.000 (0.001)
EQC	0.146 (0.040)	0.008 (0.009)	0.000 (0.002)	0.000 (0.000)	0.123 (0.036)	0.000 (0.000)
LDA	0.270 (0.050)	0.178 (0.040)	0.355 (0.073)	0.060 (0.032)	0.187 (0.043)	0.213 (0.089)
KNN	0.300 (0.055)	0.158 (0.044)	0.111 (0.041)	0.015 (0.013)	0.224 (0.045)	0.003 (0.006)
PLR	0.268 (0.048)	0.119 (0.037)	0.060 (0.027)	0.000 (0.001)	0.191 (0.041)	0.000 (0.002)
SVM	0.260 (0.053)	0.096 (0.031)	0.036 (0.016)	0.000 (0.001)	0.195 (0.040)	0.000 (0.002)
Bayes	0.186 (0.049)	0.055 (0.026)	0.020 (0.013)	0.000 (0.000)	0.154 (0.039)	0.001 (0.004)
<i>Sample size $n = 500$</i>						
DQC	0.249 (0.023)	0.076 (0.011)	0.023 (0.007)	0.000 (0.000)	0.190 (0.017)	0.001 (0.001)
Centroid	0.253 (0.022)	0.081 (0.012)	0.026 (0.008)	0.000 (0.000)	0.189 (0.018)	0.000 (0.000)
Median	0.244 (0.020)	0.063 (0.011)	0.016 (0.006)	0.000 (0.000)	0.191 (0.019)	0.000 (0.001)
CQC	0.069 (0.009)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)	0.065 (0.012)	0.000 (0.000)
EQC	0.120 (0.014)	0.004 (0.003)	0.000 (0.000)	0.000 (0.000)	0.091 (0.016)	0.000 (0.000)
LDA	0.256 (0.022)	0.091 (0.013)	0.040 (0.010)	0.356 (0.060)	0.168 (0.017)	0.000 (0.001)
KNN	0.255 (0.022)	0.126 (0.016)	0.074 (0.013)	0.005 (0.004)	0.173 (0.019)	0.004 (0.003)
PLR	0.255 (0.022)	0.097 (0.013)	0.038 (0.010)	0.000 (0.000)	0.169 (0.017)	0.000 (0.001)
SVM	0.188 (0.018)	0.076 (0.012)	0.027 (0.009)	0.000 (0.000)	0.121 (0.014)	0.000 (0.001)
Bayes	0.105 (0.016)	0.008 (0.005)	0.003 (0.003)	0.000 (0.000)	0.082 (0.013)	0.000 (0.000)