Supplementary Materials for: Multi-state quantile regression models

Alessio Farcomeni, PhD¹*  |  Marco Geraci, PhD²†

¹Department of Economics and Finance, University of Rome “Tor Vergata”
²Department of Epidemiology and Biostatistics, Arnold School of Public Health, University of South Carolina

In this supplementary material, we report additional results for the simulation study described in the paper entitled ‘Multi-state quantile regression models’.

Correspondence
Alessio Farcomeni, Department of Economics and Finance, University of Rome “Tor Vergata”, Via Columbia 2, 00133 Roma, Italy
Email: alessio.farcomeni@uniroma2.it

Funding information

Additional results for the simulation study

Log-normal errors

Here, we report root mean square error (RMSE) and bias for \( \hat{Q}(r) \) when times are log-normal. In Figures 1 and 2, we report boxplots of RMSE and bias of each estimator under the competing risk scenario, and, in Figures 3 and 4, under the multi-state process scenario.
**FIGURE 1** Root mean square error (RMSE) for $\hat{Q}(r)$ when data are generated according to a competing risks scenario and times are log-normal. Boxplots are based on $B = 1000$ replicates. MSQR: our proposed approach. CRQ: standard censored quantile regression. PF: competing risk quantile regression.
FIGURE 2 Bias for $\hat{Q}(\tau)$ when data are generated according to a competing risks scenario and times are log-normal. Boxplots are based on $B = 1000$ replicates. MSQR: our proposed approach. CRQ: standard censored quantile regression. PF: competing risk quantile regression.
**Figure 3** Root mean square error (RMSE) for \( \hat{Q}(r) \) when data are generated according to a multi-state scenario and times are log-normal. Boxplots are based on \( B = 1000 \) replicates. MSQR: our proposed approach. CRQ: standard censored quantile regression\(^1\). PF: competing risk quantile regression\(^2\).  

\(^{1}\) Censored quantile regression. \(^{2}\) Competing risk quantile regression.
**Figure 4** Bias for $\hat{Q}(r)$ when data are generated according to a multi-state risks scenario and times are log-normal. Boxplots are based on $B = 1000$ replicates. MSQR: our proposed approach. CRQ: standard censored quantile regression $^1$. PF: competing risk quantile regression $^2$. 

$\tau$: 0.1
$\gamma$: -1

$\tau$: 0.1
$\gamma$: 0

$\tau$: 0.5
$\gamma$: -1

$\tau$: 0.5
$\gamma$: 0
Weibull errors

Here, we report RMSE and bias for $\hat{\beta}(\tau)$ and $\hat{Q}(\tau)$ when times follow a Weibull distribution. In Figures 5 and 6, we report boxplots of RMSE and bias of each estimator $\hat{\beta}(\tau)$ under the competing risk scenario, and, in Figures 7 and 8, under the multi-state process scenario. In Figures 9 and 10, we report boxplots of RMSE and bias of each estimator $\hat{Q}(\tau)$ under the competing risk scenario, and, in Figures 11 and 12, under the multi-state process scenario.
**Figure 5** Root mean square error (RMSE) for $\hat{\beta}(\tau)$ when data are generated according to a competing risks scenario and times are Weibull. Boxplots are based on $B = 1000$ replicates. MSQR: our proposed approach. CRQ: standard censored quantile regression $^1$. PF: competing risk quantile regression $^2$. 
**Figure 6** Bias for $\hat{\beta}(r)$ when data are generated according to a competing risks scenario and times are Weibull. Boxplots are based on $B = 1000$ replicates. MSQR: our proposed approach. CRQ: standard censored quantile regression. PF: competing risk quantile regression.
FIGURE 7  Root mean square error (RMSE) for $\hat{\beta}(r)$ when data are generated according to a multi-state scenario and times are Weibull. Boxplots are based on $B = 1000$ replicates. MSQR: our proposed approach. CRQ: standard censored quantile regression. PF: competing risk quantile regression.
FIGURE 8  Bias for $\hat{\beta}(r)$ when data are generated according to a multi-state risks scenario and times are Weibull. Boxplots are based on $B = 1000$ replicates. MSQR: our proposed approach. CRQ: standard censored quantile regression. PF: competing risk quantile regression.
**FIGURE 9** Root mean square error (RMSE) for $\hat{Q}(\tau)$ when data are generated according to a competing risks scenario and times are Weibull. Boxplots are based on $B = 1000$ replicates. MSQR: our proposed approach. CRQ: standard censored quantile regression $^1$. PF: competing risk quantile regression $^2$. 

\[\tau: 0.1 \quad \text{gamma: } -1\]
\[\tau: 0.1 \quad \text{gamma: } 0\]
\[\tau: 0.5 \quad \text{gamma: } -1\]
\[\tau: 0.5 \quad \text{gamma: } 0\]
FIGURE 10  Bias for $\hat{Q}(r)$ when data are generated according to a competing risks scenario and times are Weibull. Boxplots are based on $B = 1000$ replicates. MSQR: our proposed approach. CRQ: standard censored quantile regression $^1$. PF: competing risk quantile regression $^2$. 
Figure 11: Root mean square error (RMSE) for $\hat{Q}(r)$ when data are generated according to a multi-state scenario and times are Weibull. Boxplots are based on $B = 1000$ replicates. MSQR: our proposed approach. CRQ: standard censored quantile regression. PF: competing risk quantile regression.
**Figure 12** Bias for $\hat{Q}(r)$ when data are generated according to a multi-state risks scenario and times are Weibull. Boxplots are based on $B = 1000$ replicates. MSQR: our proposed approach. CRQ: standard censored quantile regression $^1$. PF: competing risk quantile regression $^2$. 


REFERENCES
