ORIGINAL ARTICLE

Supplementary Materials for: Multi-state quantile regression models

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In this supplementary material, we report additional results for the simulation study described in the paper entitled 'Multi-state quantile regression models'.

| ADDITIONAL RESULTS FOR THE SIMULATION STUDY

| Log-normal errors

Here, we report root mean square error (RMSE) and bias for $\widehat{Q}(\tau)$ when times are log-normal. In Figures 1 and 2, we report boxplots of RMSE and bias of each estimator under the competing risk scenario, and, in Figures 3 and 4, under the multi-state process scenario.

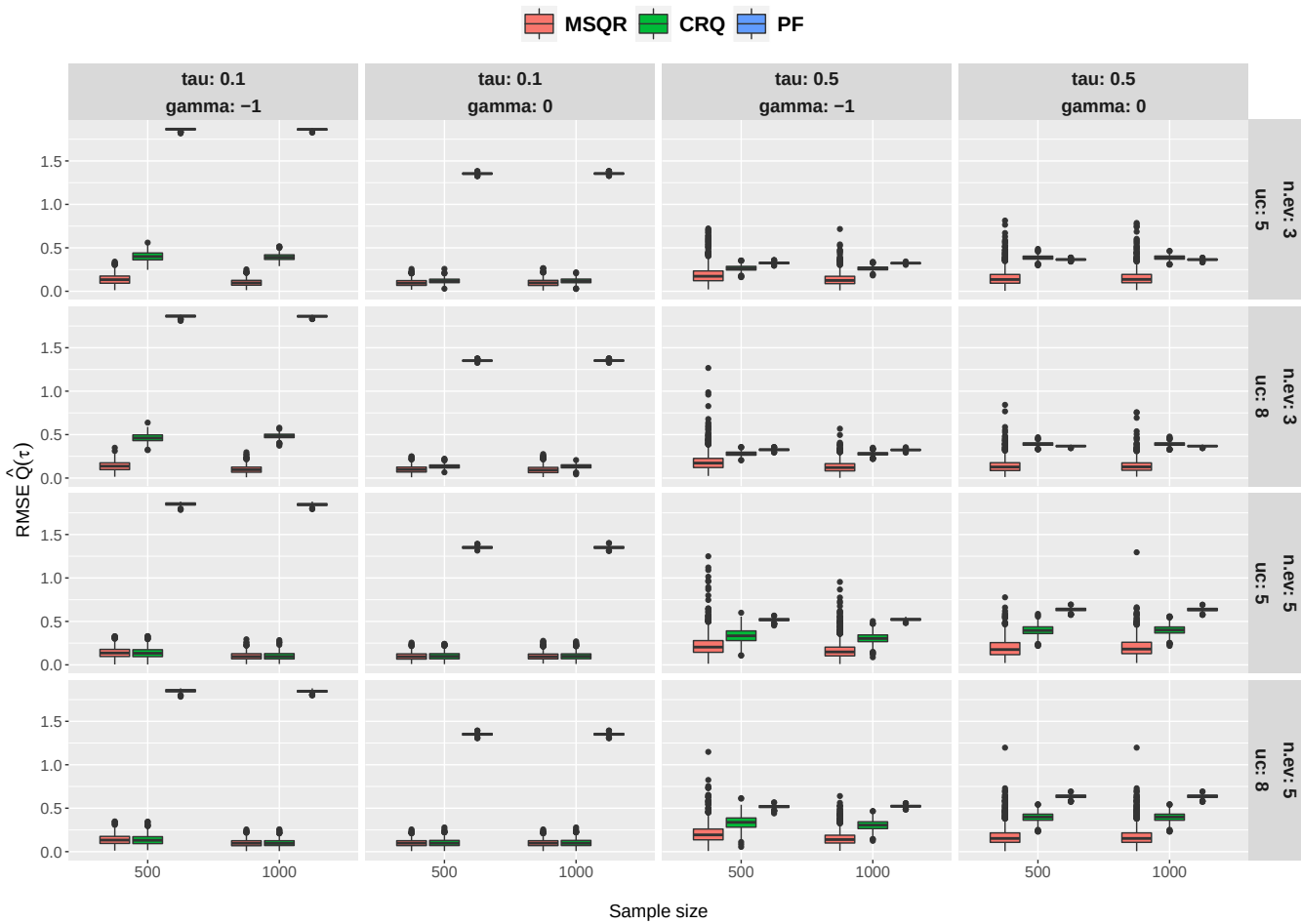

FIGURE 1 Root mean square error (RMSE) for $\widehat{Q}(\tau)$ when data are generated according to a competing risks scenario and times are log-normal. Boxplots are based on $B = 1000$ replicates. MSQR: our proposed approach. CRQ: standard censored quantile regression [1]. PF: competing risk quantile regression [2].

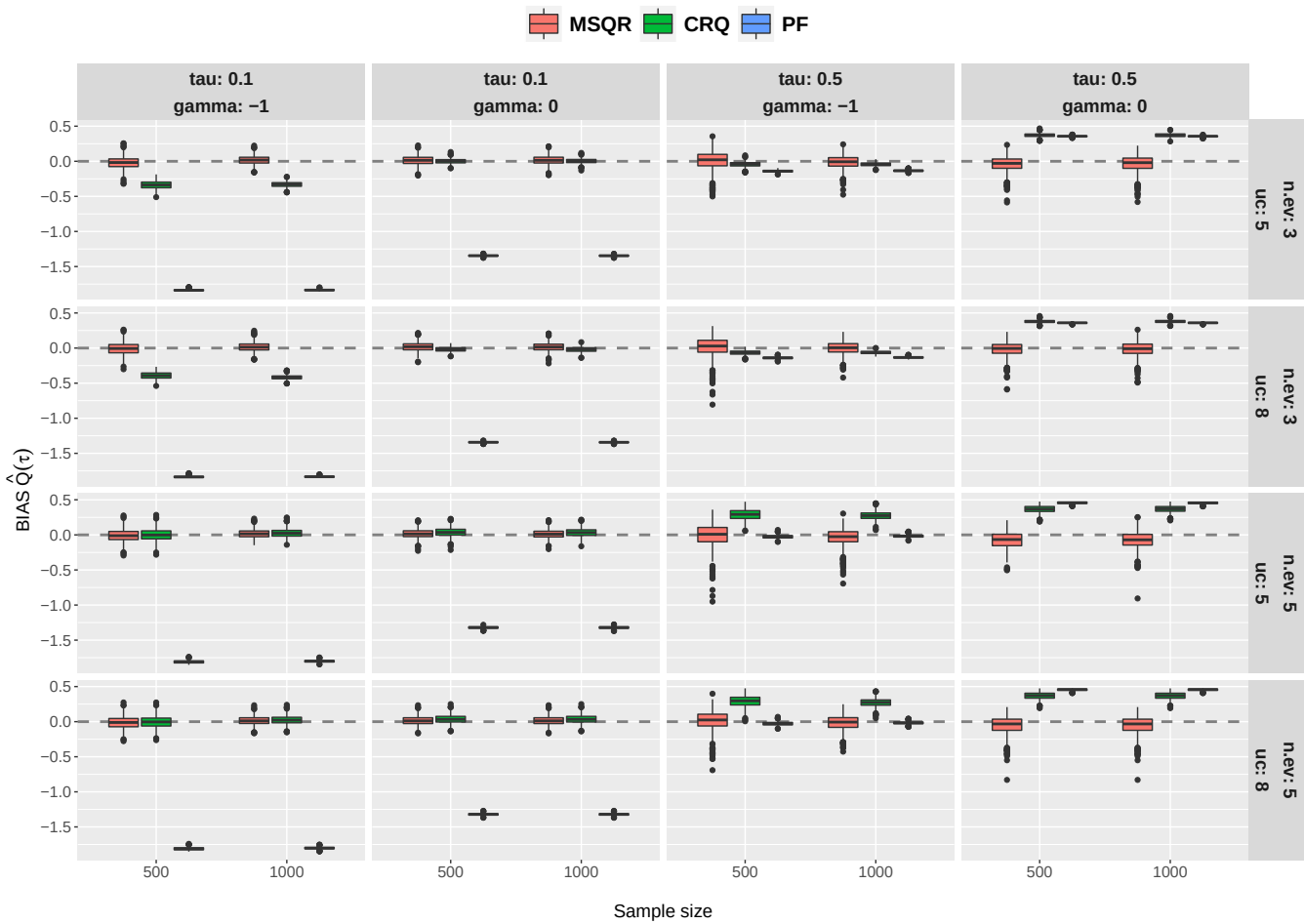

FIGURE 2 Bias for $\widehat{Q}(\tau)$ when data are generated according to a competing risks scenario and times are log-normal. Boxplots are based on $B = 1000$ replicates. MSQR: our proposed approach. CRQ: standard censored quantile regression [1]. PF: competing risk quantile regression [2].

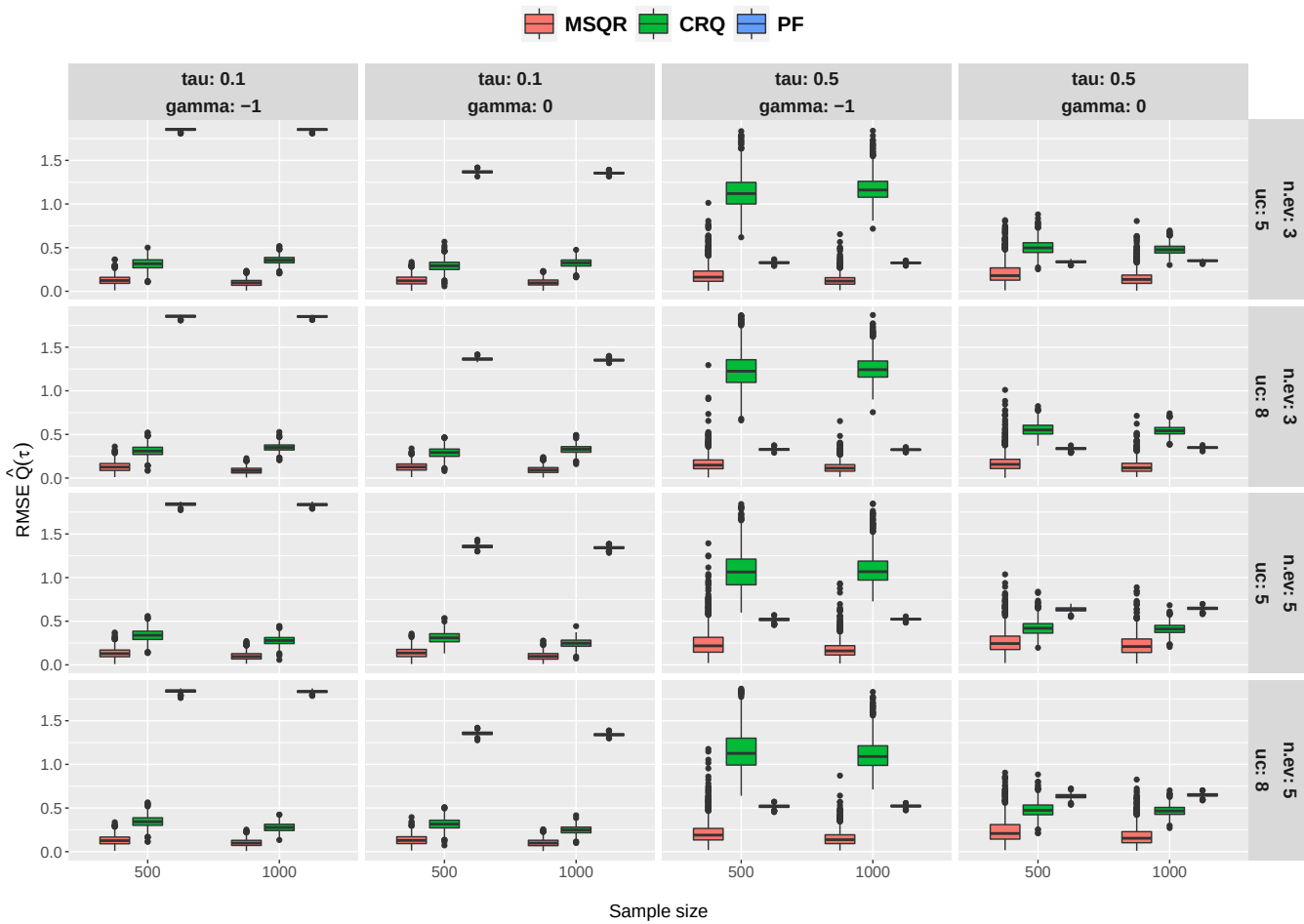

FIGURE 3 Root mean square error (RMSE) for $\widehat{Q}(\tau)$ when data are generated according to a multi-state scenario and times are log-normal. Boxplots are based on $B = 1000$ replicates. MSQR: our proposed approach. CRQ: standard censored quantile regression [1]. PF: competing risk quantile regression [2].

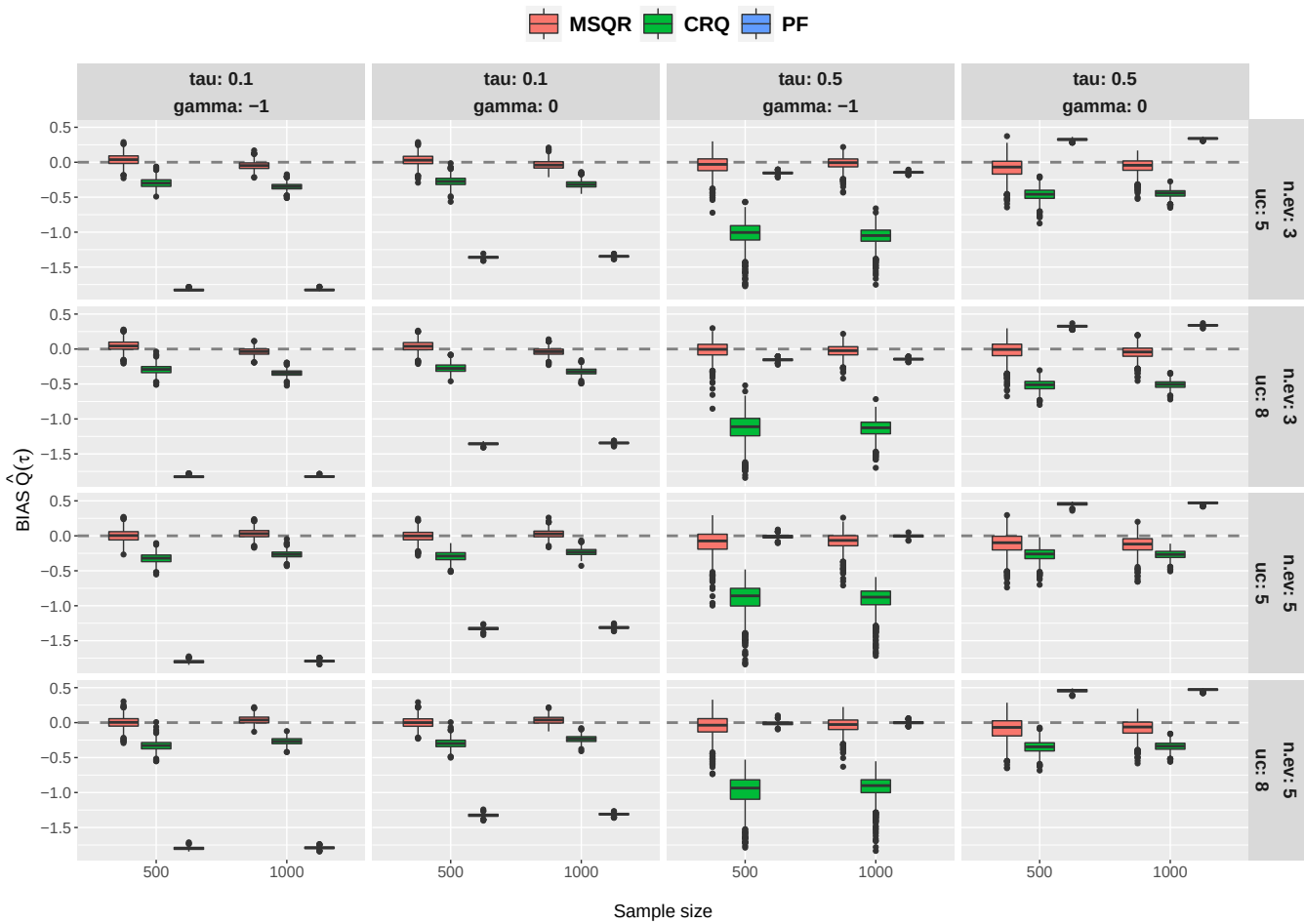

FIGURE 4 Bias for $\widehat{Q}(\tau)$ when data are generated according to a multi-state risks scenario and times are log-normal. Boxplots are based on $B = 1000$ replicates. MSQR: our proposed approach. CRQ: standard censored quantile regression [1]. PF: competing risk quantile regression [2].

| Weibull errors

Here, we report RMSE and bias for $\widehat{\beta}(\tau)$ and $\widehat{Q}(\tau)$ when times follow a Weibull distribution. In Figures 5 and 6, we report boxplots of RMSE and bias of each estimator $\widehat{\beta}(\tau)$ under the competing risk scenario, and, in Figures 7 and 8, under the multi-state process scenario. In Figures 9 and 10, we report boxplots of RMSE and bias of each estimator $\widehat{Q}(\tau)$ under the competing risk scenario, and, in Figures 11 and 12, under the multi-state process scenario.

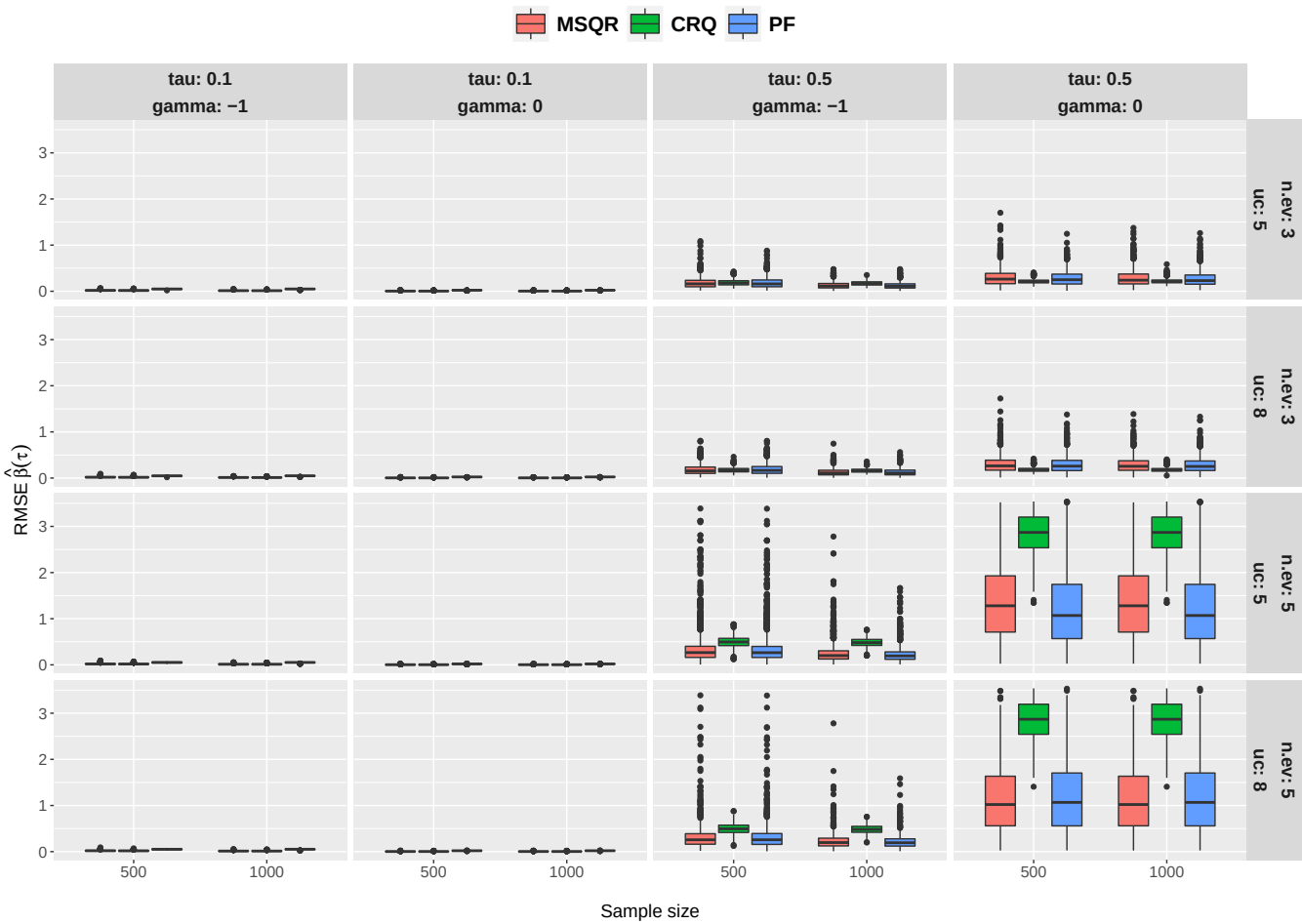

FIGURE 5 Root mean square error (RMSE) for $\widehat{\beta}(\tau)$ when data are generated according to a competing risks scenario and times are Weibull. Boxplots are based on $B = 1000$ replicates. MSQR: our proposed approach. CRQ: standard censored quantile regression [1]. PF: competing risk quantile regression [2].

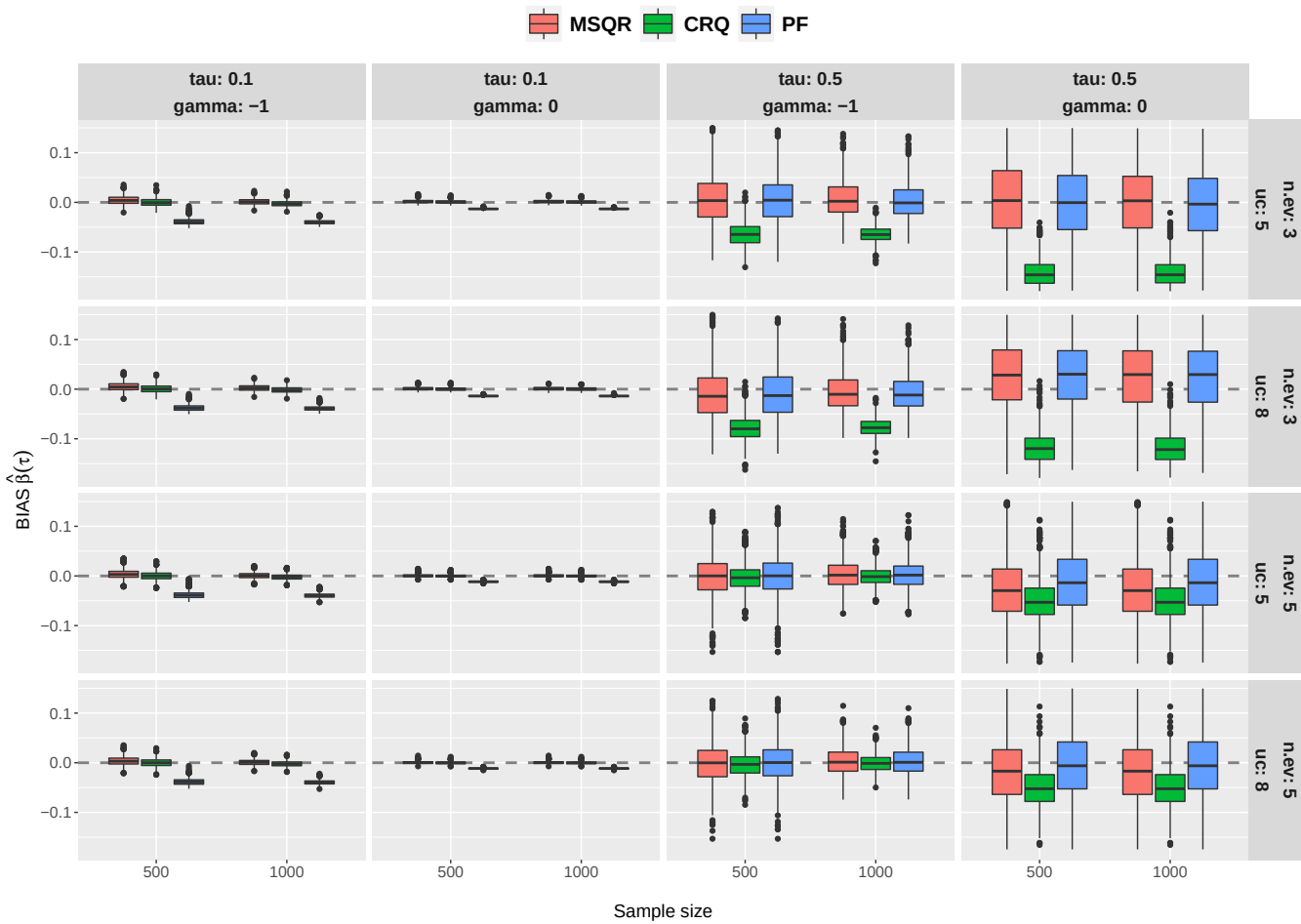

FIGURE 6 Bias for $\widehat{\beta}(\tau)$ when data are generated according to a competing risks scenario and times are Weibull. Boxplots are based on $B = 1000$ replicates. MSQR: our proposed approach. CRQ: standard censored quantile regression [1]. PF: competing risk quantile regression [2].

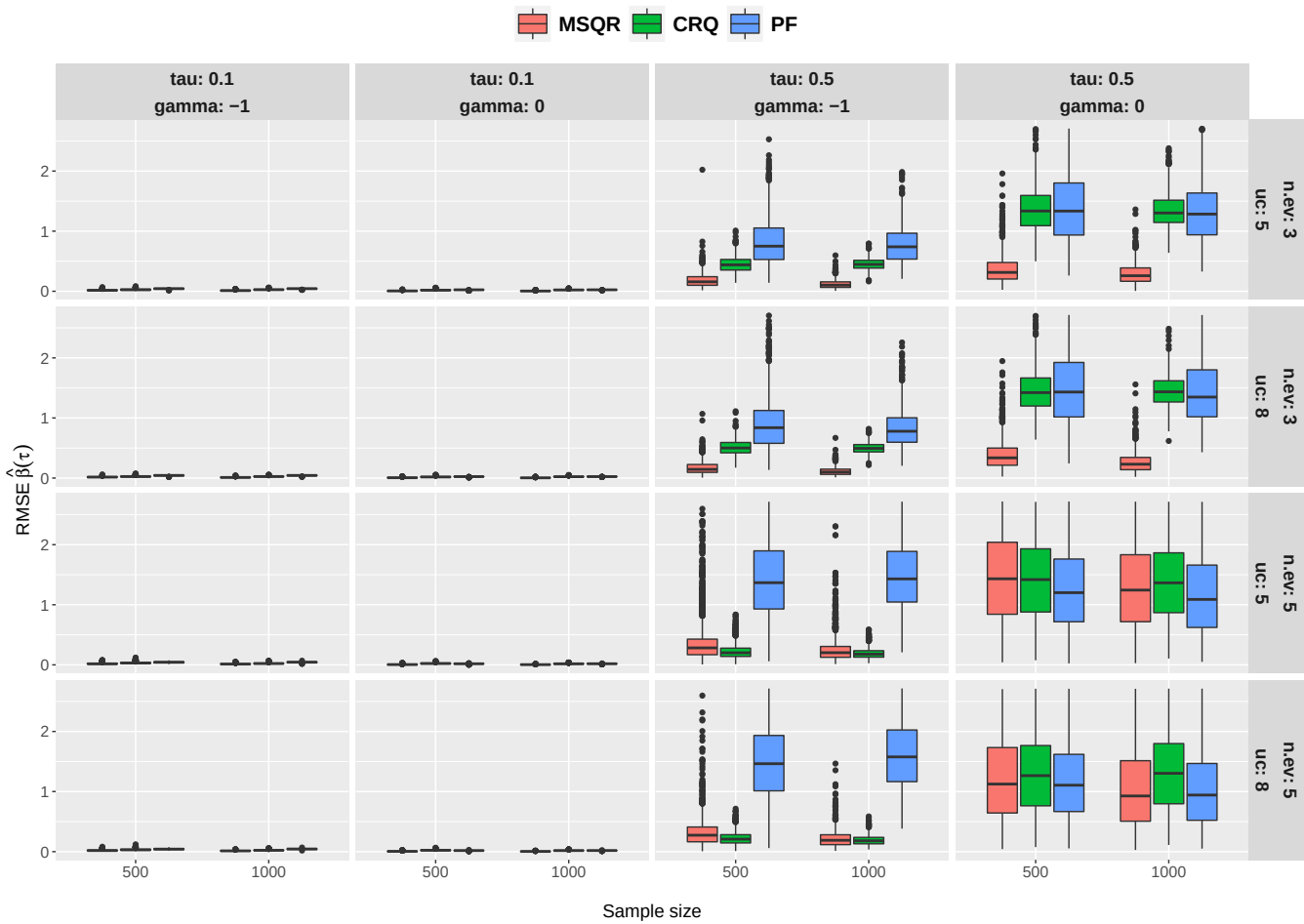

FIGURE 7 Root mean square error (RMSE) for $\widehat{\beta}(\tau)$ when data are generated according to a multi-state scenario and times are Weibull. Boxplots are based on $B = 1000$ replicates. MSQR: our proposed approach. CRQ: standard censored quantile regression [1]. PF: competing risk quantile regression [2].

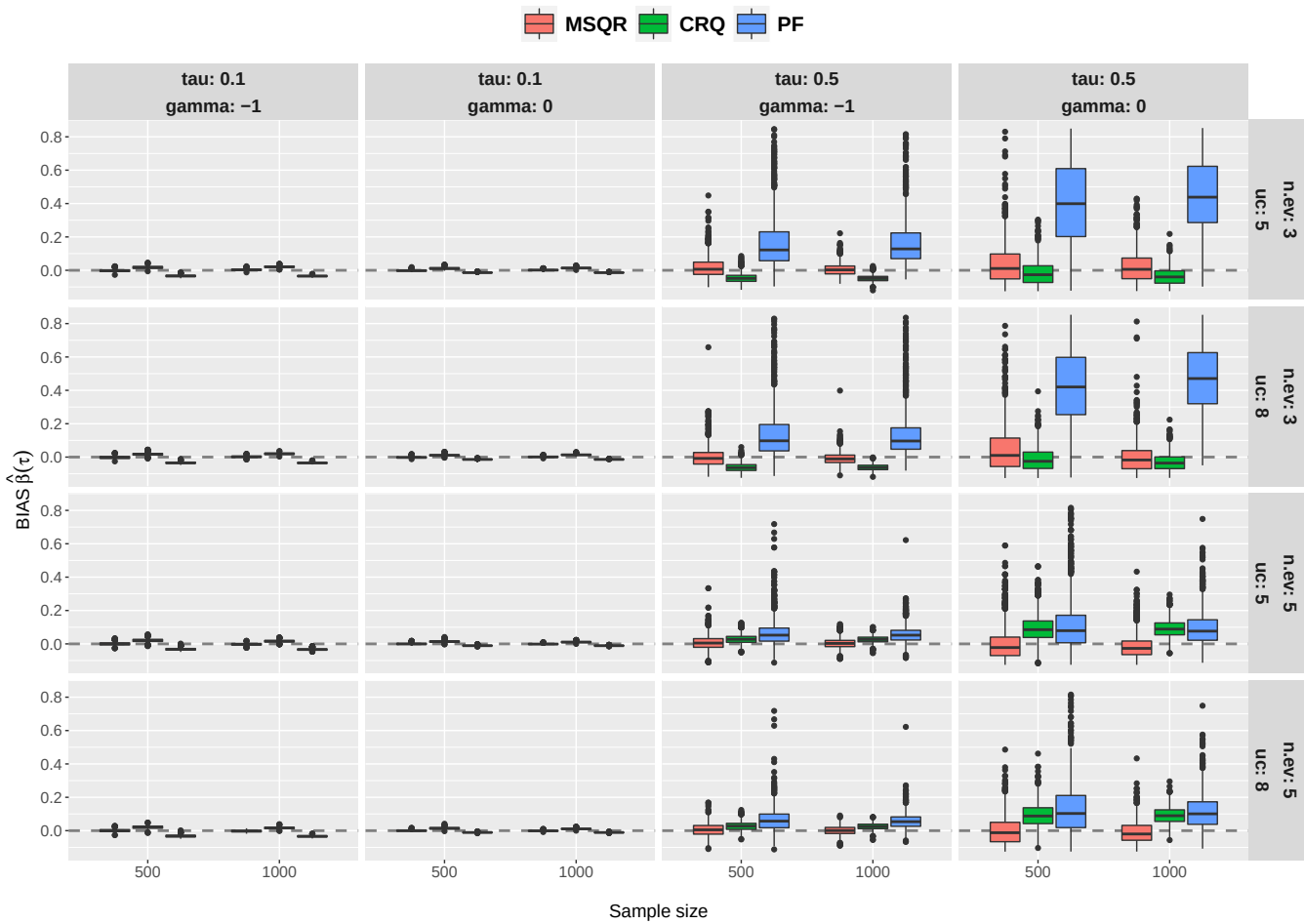

FIGURE 8 Bias for $\widehat{\beta}(\tau)$ when data are generated according to a multi-state risks scenario and times are Weibull. Boxplots are based on $B = 1000$ replicates. MSQR: our proposed approach. CRQ: standard censored quantile regression [1]. PF: competing risk quantile regression [2].

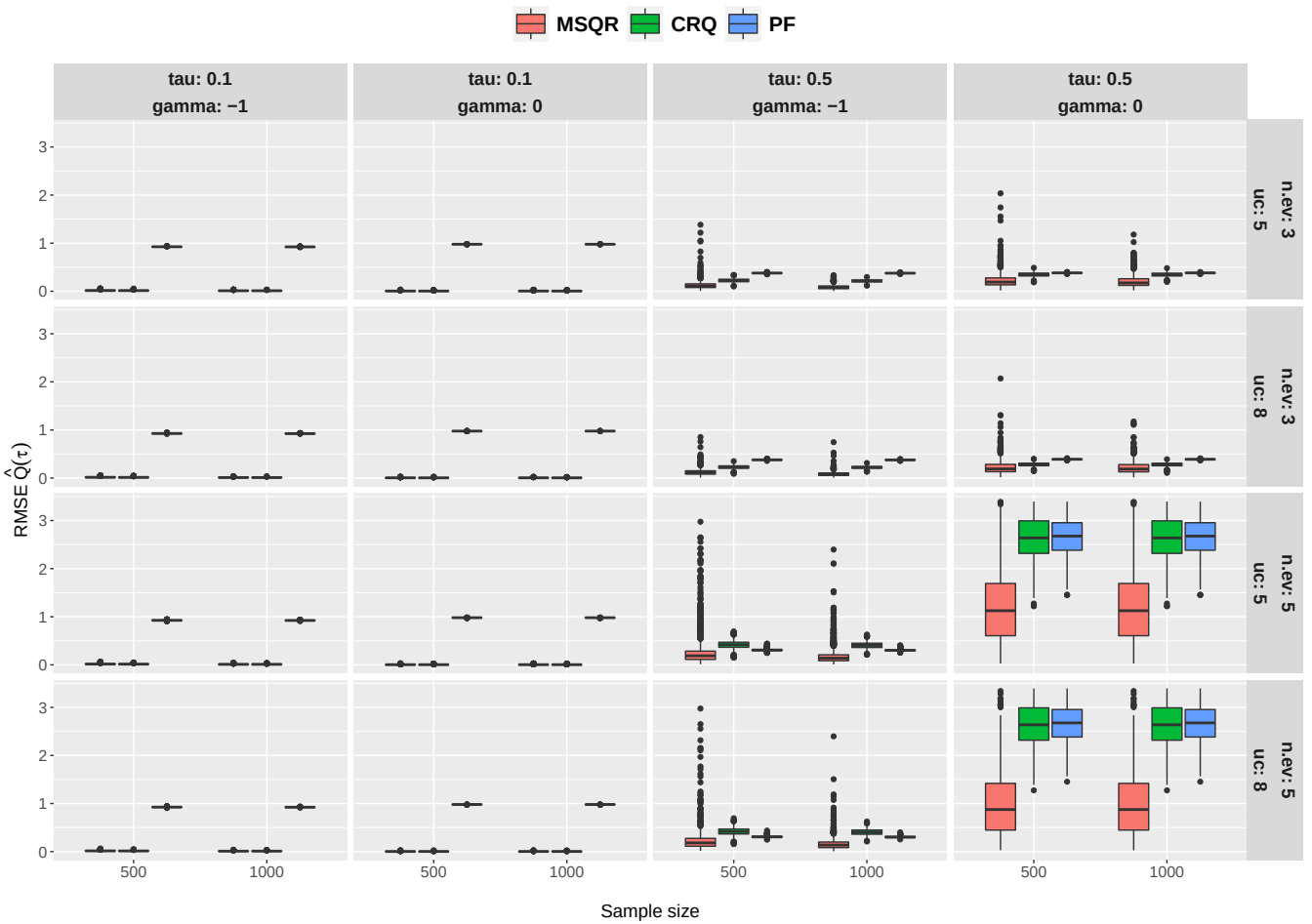

FIGURE 9 Root mean square error (RMSE) for $\widehat{Q}(\tau)$ when data are generated according to a competing risks scenario and times are Weibull. Boxplots are based on $B = 1000$ replicates. MSQR: our proposed approach. CRQ: standard censored quantile regression [1]. PF: competing risk quantile regression [2].

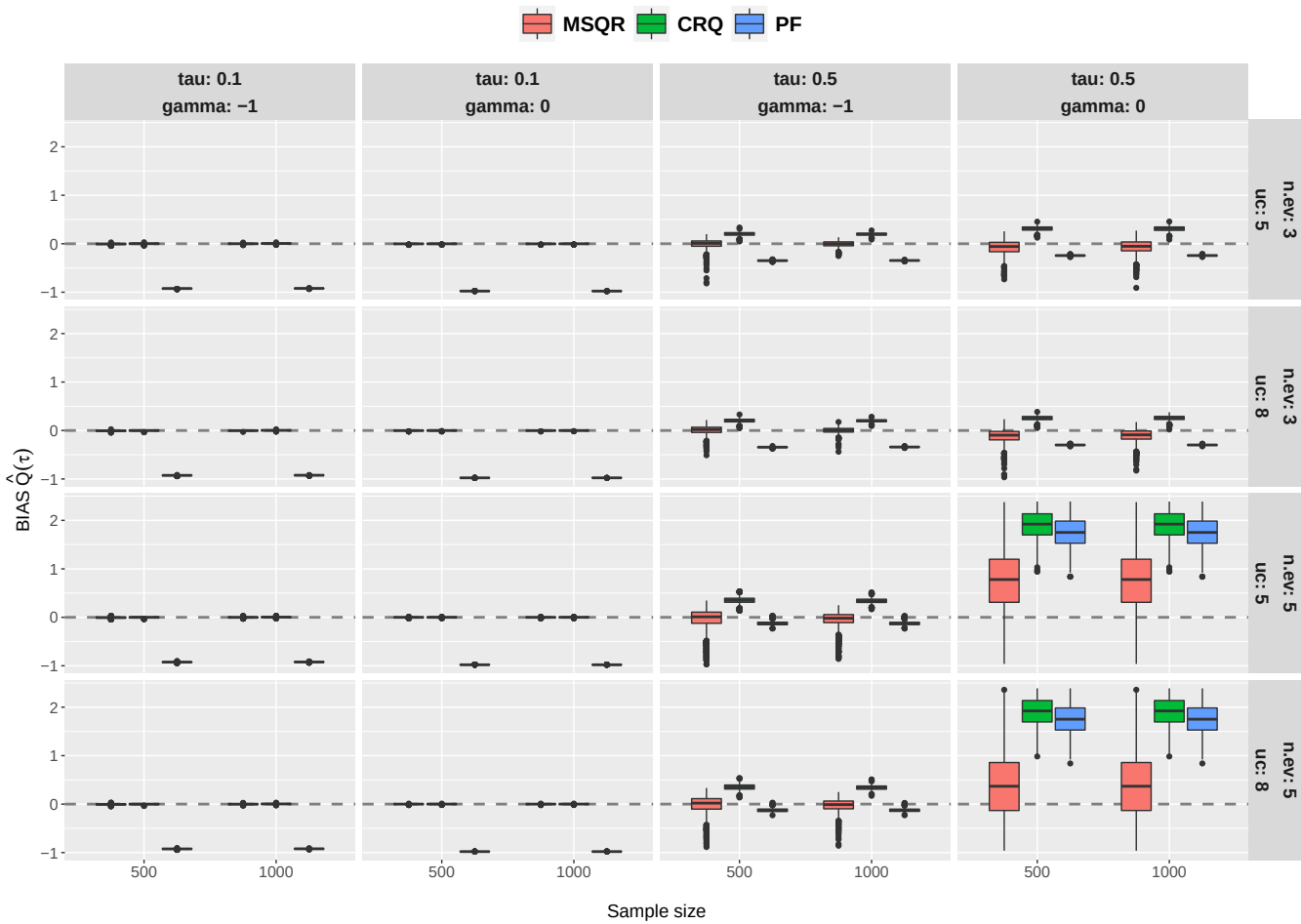

FIGURE 10 Bias for $\widehat{Q}(\tau)$ when data are generated according to a competing risks scenario and times are Weibull. Boxplots are based on $B = 1000$ replicates. MSQR: our proposed approach. CRQ: standard censored quantile regression [1]. PF: competing risk quantile regression [2].

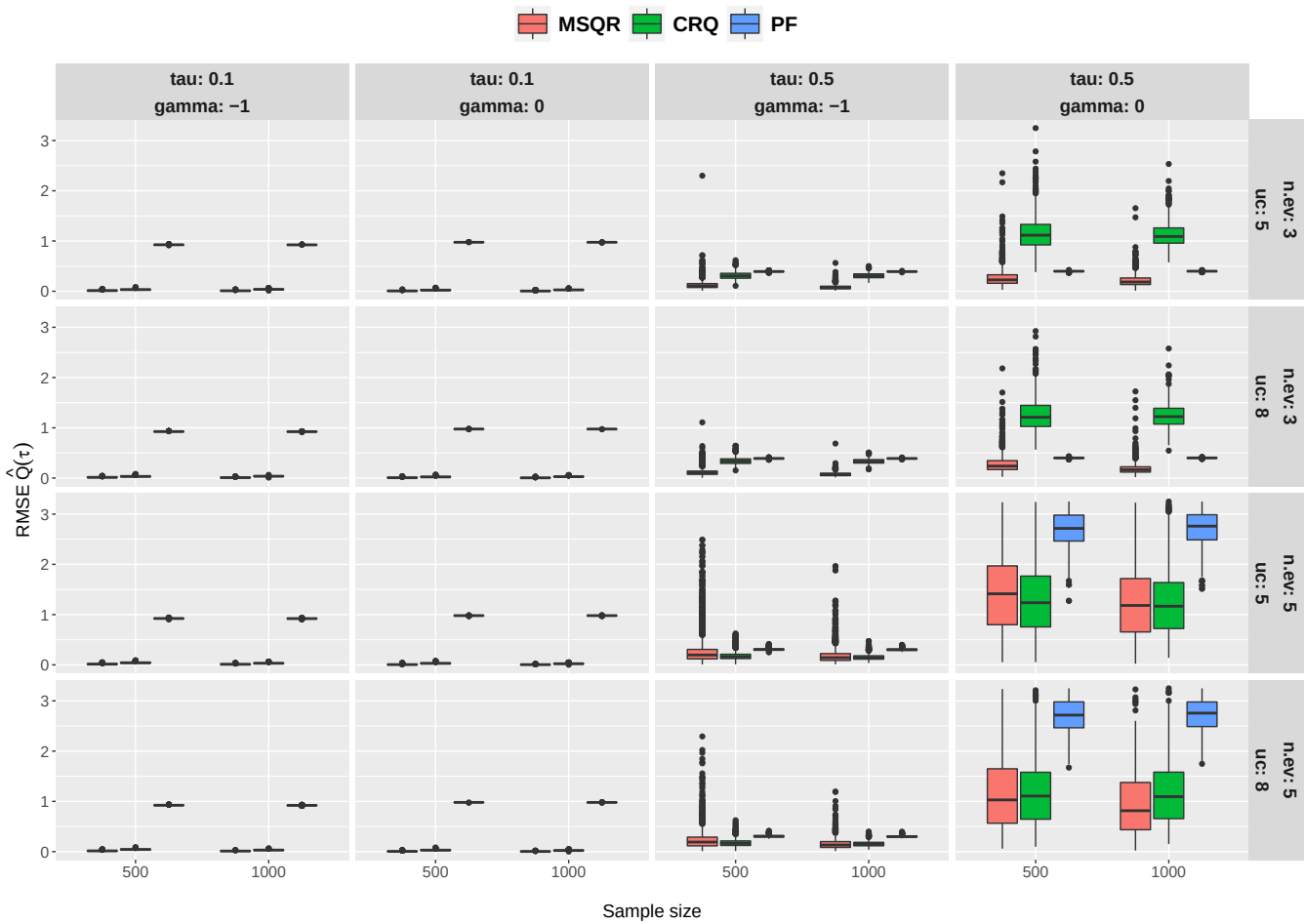

FIGURE 11 Root mean square error (RMSE) for $\widehat{Q}(\tau)$ when data are generated according to a multi-state scenario and times are Weibull. Boxplots are based on $B = 1000$ replicates. MSQR: our proposed approach. CRQ: standard censored quantile regression [1]. PF: competing risk quantile regression [2].

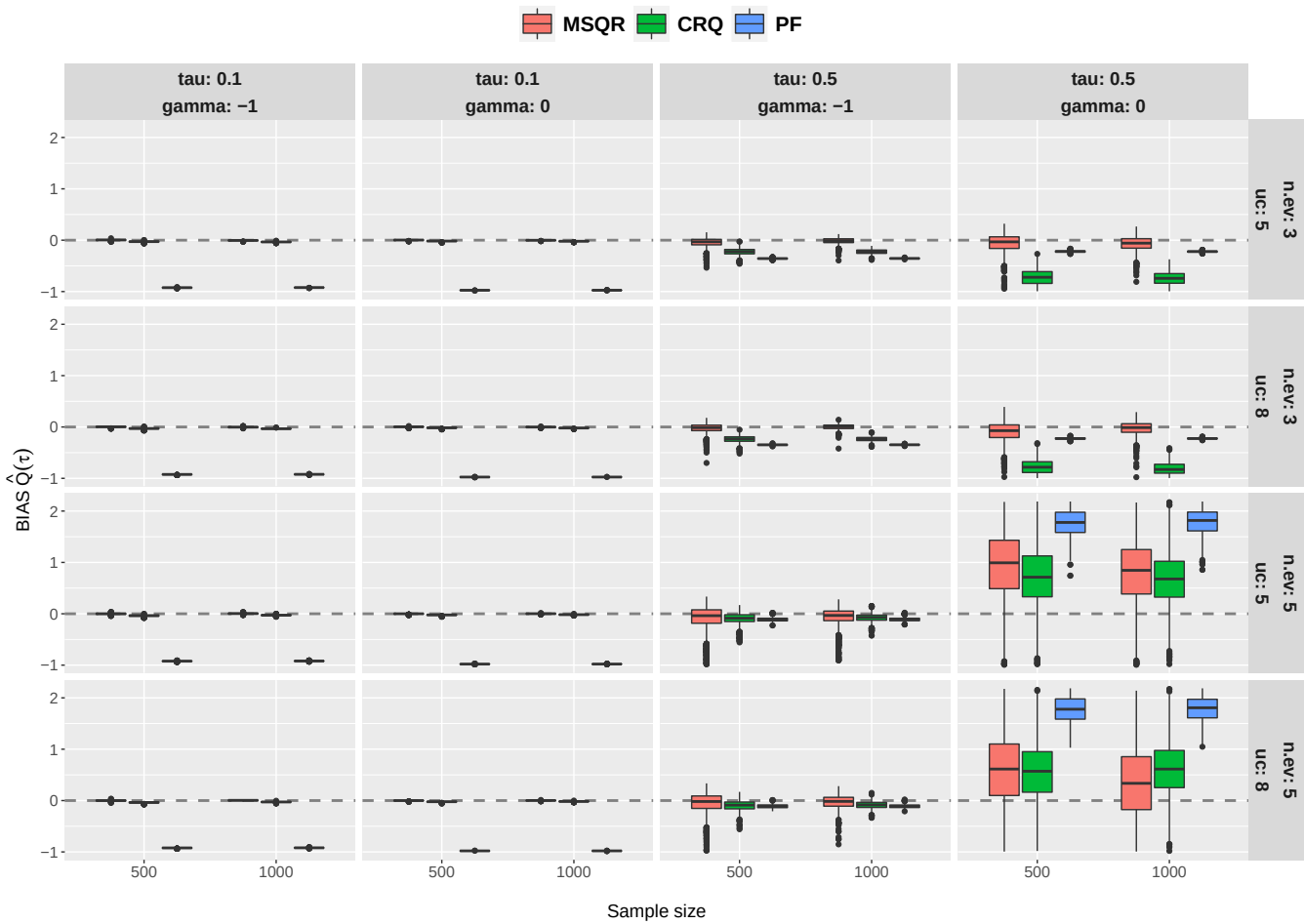

FIGURE 12 Bias for $\widehat{Q}(\tau)$ when data are generated according to a multi-state risks scenario and times are Weibull. Boxplots are based on $B = 1000$ replicates. MSQR: our proposed approach. CRQ: standard censored quantile regression [1]. PF: competing risk quantile regression [2].

REFERENCES

[1] Portnoy S. Censored Regression Quantiles. Journal of the American Statistical Association 2003;98:1001–1012.

[2] Peng L, Fine JP. Competing risks quantile regression. Journal of the American Statistical Association 2009;104:1440–1453.